

Enhanced Force Computation with the Finite Element Method

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Abstract: This paper presents a new method for computing field quantities and forces. The field variable is calculated numerically with the Finite Element Method (FEM), using the soft package MagNet V5.1. The new approach uses the finite element solution as input data and consists in solving the Laplace equation analytically, inside a local post-process. Several test problems are solved both by direct computation method and by the new approach. The implementation of the methods is made using MATLAB programs and the results are compared.

1. Introduction

The high precision numerical evaluation of the local and integral field quantities, impedances, forces, torques, etc., represents a very important problem [1], [3].

The Finite Element Method (FEM) is a well known numerical computation method. As any other numerical method, FEM gives an "approximate" solution. The main sources of inaccuracy in problems solved using FEM are [1]:

- representation of the field variables by a polynomial or other approximating function;
- representation of a continuous medium by a mesh of elements;
- errors due to computer operations;
- errors due to assumptions, idealisations.

For the magnetic forces evaluation, the following computation methods are generally used [1], [2], [3]:

- integration of Maxwell stress tensor;
- virtual displacements;
- surface force density;
- equivalent magnetising currents.

All these methods are using in computations the flux density value obtained directly from the FEM solution.

The most used force computation method is the integration of Maxwell stress tensor. The field variable (the magnetic potential vector) is obtained by numerical computations, therefore its derivative (the flux density) will be inexact, leading to an

inexact value of the force.

This paper presents an improvement of the magnetic force computation, using field solutions obtained by FEM. Solving analytically the field differential equation, inside a circular subdomain, after its discretization, a higher precision is obtained in the computation of the potential vector and its derivative.

In this paper, both the force direct computation method and the new approach, that improves the finite element approximation of the magnetic potential vector are presented.

2. Force computation using the direct method

The magnetic potential vector A , of a bidimensional stationary magnetic field in a non-linear medium verifies the following type of equation:

$$\nabla \times \left(\frac{1}{\mu} \nabla \times A \right) = J, \quad (1)$$

where:

- J represents the imposed current density value.

The computation of the total force that is acting on an arbitrary body m , from the analysed bidimensional domain, consist of the following steps:

1. Solving the magnetic field problem (1) through the FEM. If, first degree triangle elements are used, then the magnetic vector potential A at the level of an element is approximated by [1]:

$$A^e(x,y) = \sum_m N_m^e(x,y) \cdot A_m^e, \quad m = i, j, k \quad (2)$$

where:

- i, j, k are the local indexes corresponding to the "e" triangle vertices;
- A_m^e are the unknown values of the potential in the nodes of the finite element "e";
- $N_m^e(x,y)$ are the shape functions

corresponding to the first degree triangle elements.

The software package MagNet V5.1. is used to solve the proposed test problems.

2. Computing the flux density components with respect to the Ox and Oy axis:

$$B_x^e = \frac{\partial \Lambda^e}{\partial y} = \sum_m \frac{\partial N_m^e}{\partial y} A_m^e = \text{constant}, \quad (3)$$

$$B_y^e = -\frac{\partial \Lambda^e}{\partial x} = -\sum_m \frac{\partial N_m^e}{\partial x} A_m^e = \text{constant}.$$

3. Evaluation of the magnetic force using the integration of Maxwell stress tensor:

$$F_m = \int_C \frac{1}{2\mu_0} (B_n^2 - B_t^2) ds \mathbf{t} + \int_C \frac{1}{\mu_0} B_n B_t ds \mathbf{n} \quad (4)$$

where:

- C is a contour enveloping the body m in the region free of the source and magnetic material;
- B_n, B_t are the normal and the tangential components of the magnetic flux density along the contour C , respectively;
- \mathbf{t}, \mathbf{n} are the unit vector in the normal and tangential direction of the contour C .

The computation of the flux density, normal and tangential components is made using the relations (3).

This algorithm was applied to several test problems, corresponding to many magnetic devices considered to be in equilibrium. In this way it can be verified if the computed force value is null.

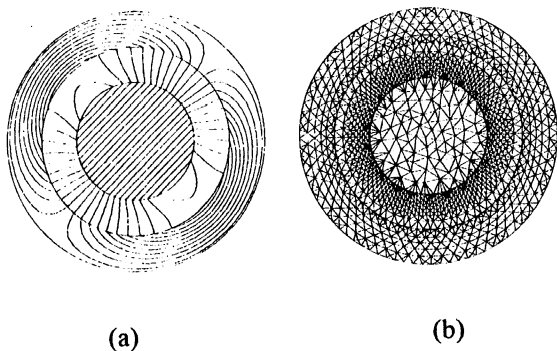


Fig.1 The flux density distribution in the analysed magnetic device (a); The FEM mesh (b).

Let us consider the structure with the radial symmetry, presented in Fig. 1, consisting of a permanent magnet inside a ferromagnetic cover. The equipotential lines and the domain discretization in triangle finite elements are

presented in Fig.1a and 1b respectively. The results obtained from the direct force computation are compared with the results obtained from the magnetic potential and flux density computation method, presented in the next paragraph.

3. The new approach

The magnetic vector potential values obtained by applying the FEM are "adjusted" within a post-processing module using the computation method presented hereafter.

Inside a circular domain of R radius, placed in the sourceless discretization domain, presented in Fig. 2, the field solution verifies a Laplace equation $\Delta A = 0$.

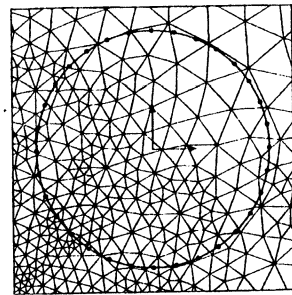


Fig. 2 The application domain of the Laplace equation

In polar coordinates, the Laplace equation is:

$$r^2 \frac{\partial^2 A}{\partial r^2} + r \frac{\partial A}{\partial r} + \frac{\partial^2 A}{\partial \vartheta^2} = 0. \quad (5)$$

Equation (5), solved by using the variable separation method leads to a solution of the following form, [3], [4], [5]:

$$A(r, \vartheta) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\vartheta + b_n \sin n\vartheta) r^n \quad (6)$$

The coefficients a_n and b_n are obtained from the potential at the circumference $f(\vartheta) = A(R, \vartheta)$ and are the Fourier coefficients of the period boundary function:

$$a_n = \frac{1}{\pi R^n} \int_0^{2\pi} f(\vartheta) \cos n\vartheta d\vartheta$$

$$b_n = \frac{1}{\pi R^n} \int_0^{2\pi} f(\vartheta) \sin n\vartheta d\vartheta. \quad (7)$$

It is worth mentioning that for the condition on the $f(\vartheta)$ boundary, there is no any available analytical expression. Solving the field problem using FEM, the magnetic vector potential values in the discretization mesh nodes, situated inside the

circular domain are determined. Then the potential values corresponding to a finite number of elements placed on the circular boundary, are computed by interpolation, using different types of functions specific to each finite element.

For a finite number N of equidistant spaced points $A_k=f(\vartheta_k)=f(2k\pi/N)$, discrete transformation can be used:

$$\begin{aligned} a_0 &= \frac{1}{N} \sum_{k=1}^N A_k \\ a_n &= \frac{2}{NR^n} \sum_{k=1}^N A_k \cos(n\vartheta_k) \\ b_n &= \frac{2}{NR^n} \sum_{k=1}^N A_k \sin(n\vartheta_k) \\ \vartheta_k &= \frac{2k\pi}{N} \end{aligned} \quad (8)$$

With the a_n and b_n constants determined as above, an analytical expression for the magnetic vector potential is obtained. Only the potential A_k at the circumference $r = R$ have to be known. Good results are obtained with $N = 30$ points on the circle.

The analytical expression of the magnetic vector potential (6) in trigonometric form is then transformed, through successive operations in an algebraic form. Maintaining only the first 5 terms of the expression, the following expression is obtained:

$$A(x, y) = \frac{a_0}{2} + a_1x + b_1y + a_2(x^2 - y^2) + b_2 2xy + ..$$

where:

- a_0, a_1, a_2, b_1 and b_2 are the Fourier coefficients computed with the relation (8);
- x, y are local coordinates, with respect to the circular domain considered and are equal with:

$$\begin{aligned} x &= x - x_c \\ y &= y - y_c \end{aligned}$$

(x, y) are the global coordinates of a current point, while (x_c, y_c) are the global coordinates of the circle's centre with the R radius, that represents the domain boundary.

Using the expression $A(x,y)$ obtained before, the computed flux density components in the centre of the circle with R radius become:

$$B_x = \left(\frac{\partial A(x,y)}{\partial y} \right)_{\substack{x=0 \\ y=0}} = b_1$$

$$B_y = - \left(\frac{\partial A(x,y)}{\partial x} \right)_{\substack{x=0 \\ y=0}} = -a_1 \quad (9)$$

Using the direct computation for obtaining the flux density components (3), a constant value is obtained for each finite element.

The new approach ensures the continuity of the flux density components on the separation surface between the finite elements. In order to make a comparison, in Fig. 3 and Fig. 4, the distribution of the B_y flux density component, inside the same circular subdomain computed by these two methods are presented.

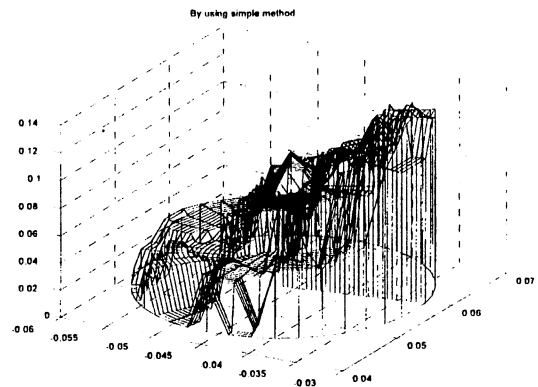


Fig. 3 Flux density B_y computed with the direct FEM

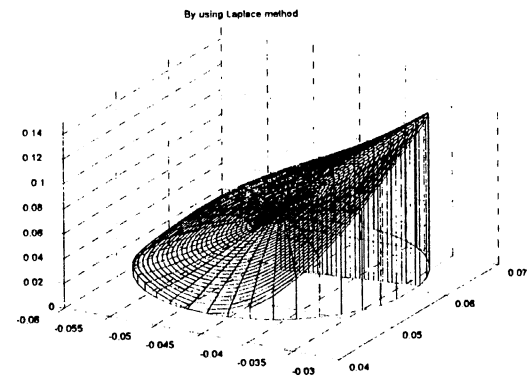


Fig. 4 Flux density B_y computed with the new approach

4. Results

Hereafter a succession of comparative results obtained for the same analysed field structure are presented. In Figs. 5 and 6, the B_n and B_t components, on a circular integration contour C , situated in the air gap are shown. For each component the corresponding values are computed using both methods.

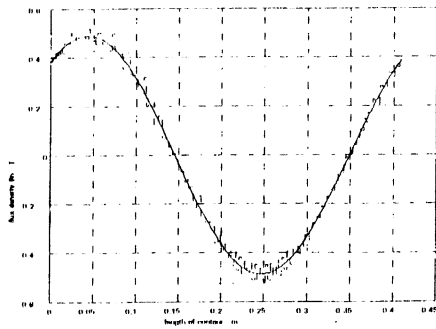


Fig. 5 The flux density B_n on the integration contour

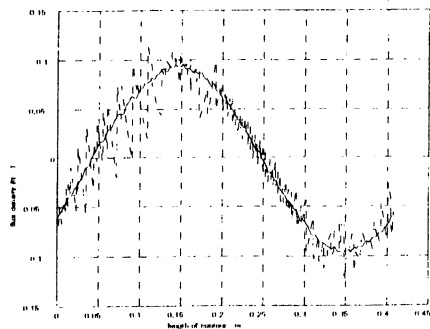


Fig. 6 The flux density B_t on the integration contour

It can be mentioned that, using the new approach both flux density components become smoother along the whole integration contour. Evident, this effects the value obtained for the force.

The force components were evaluated by successive modifications of some parameters in order to permit a better observation of their influence upon the obtained results. The parameters taken into consideration were:

- the number of the points situated on the integration contour. It can be observed that due to the relative small domain dimensions, the increase in the number of the points situated on the contour does not influence the force value F_t , computed with the new approach, Fig.7.
- the radius R of the circle, inside which the Laplace equation is applied. The radius must be correlated with the number (N) of the points on the circle. The bigger the radius is, the higher the number of the points situated on the circle is, due to the increase in the number of the finite elements "crossing" the circle.

It can be observed from Fig. 8 that the computation precision of the force F_t corresponding to the new approach is higher by rising the R radius. The proposed algorithm was verified also on other test problems and the results were identical.

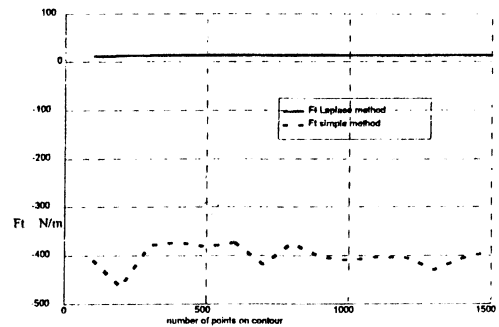


Fig. 7 F_t dependence on the number of points on the contour of integration

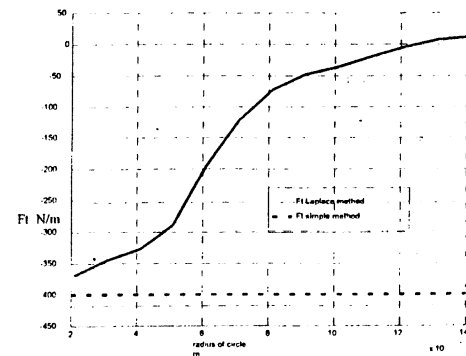


Fig. 8 F_t dependence on the radius of a circle

In conclusion, it has been shown that the accuracy of FEM approximation of field quantities and forces can be significantly improved by a post-processor operator. The proposed method is stable and easily computable for the Laplace equation.

5. References

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