

Analysis of a linear motor by a nonlinear complex formulation

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Abstract. This paper presents a finite element analysis with complex variables of nonlinear devices. The model is used for the computation of global values such as forces or Joule effect. The principle of this approach is a simplified version of the harmonic balance method. The example of a linear induction motor is presented, a special interest is paid on the computation of the electromagnetic forces, the influence of the frequency is shown up and discussed.

1. Introduction

In harmonic problems, a finite element modelling intends to give only global values (such as forces or Joule effect in the system), local values of the fields are of secondary importance.

This paper presents a method to compute accurately those kinds of global values for nonlinear systems.

The principle of the method is a simplified version of the harmonic balance method [1] in which only the first harmonic is retained. That implies that a purely sinusoidal solution of the problem is assumed in spite of the nonlinearity. That also imply that one has to determine a scalar equivalent reluctivity ν_{eq} [2] [4] so that an harmonic induction field \vec{B} will give an harmonic magnetic field \vec{H} that can be considered, according to some criterion, as equivalent to the actual periodic but non harmonic magnetic field.

The problem is then to choose a suitable criterion (for example: conservation of the magnetic energy) in order to build up an equivalent magnetization curve. Different criteria will give different curves. The originality of this method lies also in the nontrivial adaptation of the Newton-Raphson scheme to systems with complex variables [2].

2. Complex formulation

The basis of this method is to suppose that the wave forms of both magnetic field and magnetic flux density remain sinusoidal even if the magnetic materials are nonlinear. This is expressed by an equivalent constitutive law of the form $\vec{H} = \nu_{eq}(\vec{B}_{max})\vec{B}$ where the components of both \vec{H} and \vec{B} are complex numbers. This law has to be deduced from the first magnetization curve by defining an equivalent magnetic reluctivity (see §4). A possibility is to consider that H and B have the same phase angle and that the modulus of B depends on the modulus of H according to the equivalent magnetization curve. Instead of the modulus, the RMS value may be considered [2] [4]. Maxwell's equations for harmonic problems taking eddy currents into account can be written as follows:

$$\vec{\nabla} \times (\nu \vec{\nabla} \times \vec{A}) = -\sigma (j\omega \vec{A} + \vec{\nabla}V) \quad , \quad (1)$$

where ν is the reluctivity of the material, \vec{A} is the magnetic vector potential, V is the electric scalar potential, σ is the electric conductivity of the material and ω the pulsation. Two-dimensional plane and axisymmetrical problems are considered. By using Galerkin's

method, where the trial and shape functions are the same (α_i), the weak formulation of (1) for the domain Ω of boundary Γ can be expressed by :

$$\sum_{k=1}^N \left(j\omega \int_{\Omega} \sigma \alpha_i \alpha_k d\Omega + \int_{\Omega} v(A) \mathcal{B}(\alpha_i) \cdot \mathcal{B}(\alpha_k) d\Omega \right) A_k - \int_{\Omega} \sigma \alpha_i U d\Omega - \int_{\Gamma} v \alpha_i H_t d\Gamma = 0, \quad i = 1 \text{ to } N. \quad (2)$$

where

$$\mathcal{B}(\alpha_i) = \left\{ \frac{\partial \alpha_i}{\partial y}, -\frac{\partial \alpha_i}{\partial x} \right\} \text{ for the plane case.} \quad (3)$$

The boundary term in (2) including the H_t unknown can be used to apply either a nonhomogeneous Neumann boundary condition or a tangential magnetic field continuity condition at the interface between two finite element subdomains [3].

These equations are nonlinear because v is a function of A field and consequently of the A_i 's, i.e. the nodal values of the vector potential. The Newton-Raphson algorithm is used to solve these equations with an approximated Jacobian matrix [2] defined as follows :

$$J_{ik} = M_{ik} + \int_{\Omega} 2 \frac{\partial v}{\partial |B|^2} \Re \left[(\mathcal{B}(\alpha_i) \cdot \bar{B}^*) (\mathcal{B}(\alpha_k) \cdot \bar{B}) \right] d\Omega \quad (4)$$

where

- $\frac{\partial v}{\partial |B|^2}$ denotes the derivative of the reluctivity with respect to the value of the squared modulus induction;
- $\Re[\text{expression}]$ denotes the real part of the expression in brackets;
- \bar{B}^* denotes the conjugate value of the induction.

It has the interesting property to come down to the usual real variable expression of the Jacobian matrix if $\bar{B}^* = \bar{B}$ (\bar{B} is real).

3. Forces and torques

The *Maxwell stress tensor method* suits very well the finite element subdomains [3] because H_t is directly computed on all the boundaries in that method and the normal induction B_n is simply computed from the vector potential by :

$$B_n = (\bar{\nabla} \times \bar{A}) \cdot \bar{n} = -\frac{\partial A}{\partial \xi} / J(\xi) \quad (5)$$

where ξ is the local coordinate of the form functions used for the boundary element discretization and $J(\xi)$ the corresponding jacobian.

This method allows to define on the boundary an equivalent normal force density F_n and an equivalent tangential force density F_t as :

$$\bar{F}_n = \left(\frac{B_n^2}{2\mu_0} - \frac{\mu_0}{2} H_t^2 \right) \bar{n}; \quad \bar{F}_t = B_n H_t \bar{t} \quad (6;7)$$

The total force acting on an object of boundary Γ and the total torque are then given by

$$\bar{F} = \oint_{\Gamma} (\bar{F}_t + \bar{F}_n) d\Gamma; \quad \bar{T} = \oint_{\Gamma} \bar{r} \times (\bar{F}_t + \bar{F}_n) d\Gamma \quad (7;8)$$

4. Determination of the Equivalent Reluctivity

The determination of the equivalent reluctivity [2] is based on the first magnetization curve. In fact, both magnetic field and magnetic flux density are represented by vector

phasors. The new magnetization curve is obtained as follow: for each value of B_{max} (the great axis of the ellipse described by the vector \vec{B} in the plane (x,y) during a period), we compute H_{eq} , so that the magnetic energy corresponding to B_{max} is equal to $1/2 H_{eq} B_{max}$. This leads to take a constant value of the reluctivity along a period defined as :

$$\nu_{eq} = \frac{2}{B_{max}^2} \int_0^{B_{max}} H dB \quad (10)$$

5. Results

A single sided induction linear motor guiding a steel plate is considered. The movement of the plate is parallel to the wires, so perpendicular to the acting force, but the plate can accidentally deviate laterally. The force is expected to bring it back in its central position. A cross section is used as 2D model. The frequency behaviour is analysed in order to point up the optimal conditions : a maximum lateral force and a minimum vertical force. Indeed, the last must be minimised to avoid damages due to contact between the plate and the yoke. The motor is composed of one pair of poles and 12 slots in a yoke made of linear steel ($\mu_r = 2000$). Each slot is filled in with 100 copper wires. The power supply is composed of three-phased current sources that inject $10/\sqrt{2} A_{rms}$. The current distribution leads to a quasi harmonic magnetic flux density wave going from the right to the left in the air gap. Figures 1, 2 and 3 show the slip of the magnetic field lines each $\omega t = 60^\circ$: the spatial lag corresponds to 2 slots.

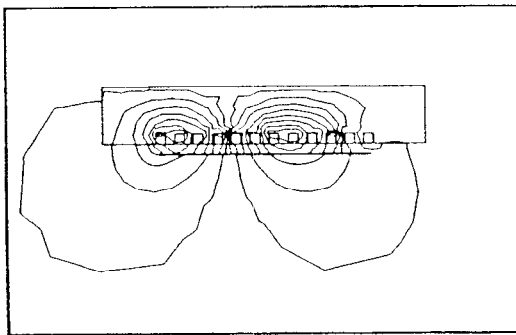


Fig.1. Phase 0°

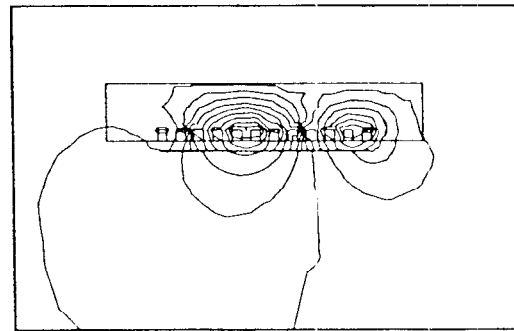


Fig.2. Phase 60°

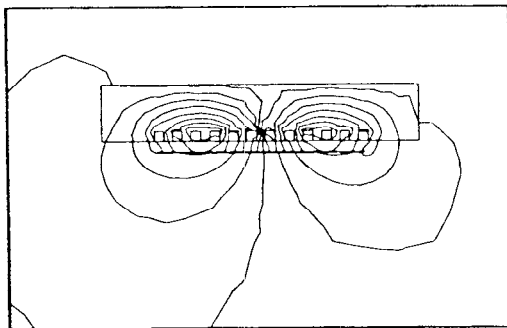


Fig.3. Phase 120°

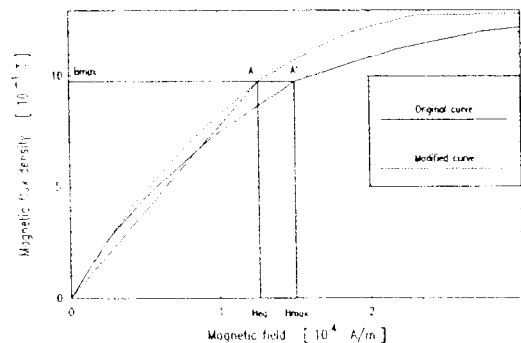


Fig.4. Saturation curves

We are now in position to make the analysis of the linear motor. The plate is made of saturable steel. An equivalent reluctivity ν_{eq} is computed with (10). The original and the modified first magnetization curves are represented in Fig. 4, and we can observe on this figure that the surface of the triangular area (O,A, B_{max}) is equal to the surface of the area (O,A', B_{max}) on the left of the first magnetization curve which is the magnetic energy density corresponding to B_{max} .

On the other hand, because \bar{H} and \bar{B} are harmonic, so are H_t and B_n . The force computed by (7) presents a mean component and an oscillating one. Only the first one is of interest in our case.

As in rotating induction motor, the magnetic induction wave induces eddy current in the plate that behave like magnetic dipoles. The interaction between poles of the plate and the stator is concretised by a resulting force, composed of an horizontal and a vertical component. The horizontal force is always positive (to the left) because the plate 'tries' to follow the magnetic field wave. The vertical component is due to two phenomena: the first is an attractive force that try to close the air gap in order to minimise the magnetic circuit reluctivity; the second is a repulsive levitation force due to dynamic phenomena, i.e. the repulsion of the magnetic dipoles. The latter one is masked at low frequency but can become preponderant at high frequency. Fig. 5 shows a plot of horizontal and vertical forces as a function of the frequency. It appears that both optimal criteria are satisfied at the same frequency (≈ 800 Hz).

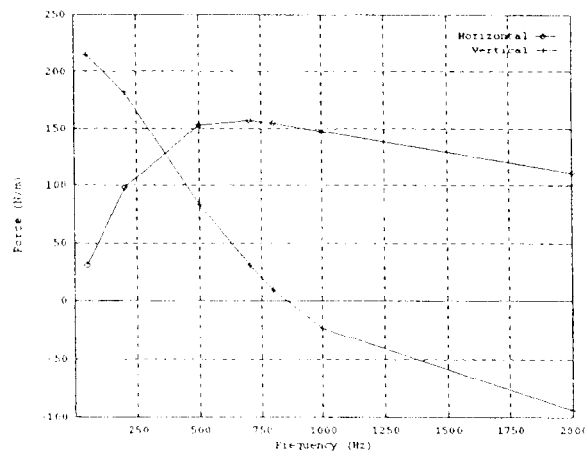


Fig. 5. Evolution of the force with frequency.

6. Conclusion

The proposed method gives a way to approximate global values such as electromagnetic forces or Joule losses in harmonic nonlinear systems. Electromechanical coupled problems can be solved in a simply way, for instance the determination of the torque as a function of the slip in asynchronous motors.

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