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A 2D h-formulation with current in the cross section and application to lightning impact on structures

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Abstract. A 2D magnetic field formulation is described that is used to analyse lightning current pulse impacting anisotropic and/or nonlinear aeronautic structures. The Newton-Raphson method associated with an under-relaxation factor is used to solve nonlinear equations. An application to panel connection is presented with the computation of heat losses. The dislocation phenomenon in metallic junctions is highlighted.

1. Introduction

A modelling of lightning impact in anisotropic and nonlinear 2D structures is presented. Experimental investigations [1] have shown that the lightning current pulse can be accurately described by a bi-exponential curve: first, the current increases drastically during a few microseconds, then it decreases slowly and tends towards a constant value after some milliseconds.

A 2D magnetic field formulation (**h**-formulation) [2] is developed to model the current density distribution. This density is assumed to flow in the studied cross section. In that case, the unknown magnetic field reduces to one component perpendicular to this section. The boundary conditions expressed in term of the magnetic field or its normal derivative are respectively equivalent to conditions relative to the normal or to the tangential component of the current density.

Planar thin structures are studied, but the problem can be extended to limit cases of thin tubular shaped axisymmetrical structures where the thickness is small in comparison with the mean radius. The distinction will be made by the way the boundary conditions are imposed.

The numerical model which has been developed enables to consider saturable materials, such as steel. This introduces nonlinear equations which are solved using a modified Newton-Raphson scheme with a relaxation factor [3] to ensure a better convergence. Anisotropic conductors, such as carbon epoxy, lead to the introduction of a resistivity tensor in the local Ohm law.

An application to fuselage panel connection impacted by a lightning pulse is presented. Particular influence of anisotropic materials on the solution is shown up and repulsion between panels, which causes dislocation phenomenon, is pointed out.

2. Magnetic field formulation

The aim is to find the transient evolution of the current density in the cross section of a conductor. Since the magnetic phenomena are not negligible (i.e., the time derivative of the magnetic flux density cannot be neglected in the Faraday law), the problem does not belong

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to electrokinetics but to magnetodynamics. So, the Faraday law (1) and the Ampere law (2) have to be solved, taking into account the local Ohm law (3) and the magnetic material behaviour (4). The conductors are expected to be electrically anisotropic, which is taken into account by a tensorial resistivity in (3). The magnetic constitutive law (4) can be nonlinear.

$$\text{curl } \mathbf{e} = -\partial_t \mathbf{b} , \quad \text{curl } \mathbf{h} = \mathbf{j} , \tag{1-2}$$

$$\mathbf{e} = \underline{\rho}\, \mathbf{j} , \quad \mathbf{b} = \mu(h)\, \mathbf{h} . \tag{3-4}$$

The current density that flows in the cross section of a conductor has two components, along x and y axis. The magnetic field is reduced to one component along z axis because of the 2D nature of the structure.

The Ampere law applied in the nonconductive surrounding media, i.e. $\text{curl } \mathbf{h} = 0$, ensures the magnetic field to be constant in space, but not necessarily in time. Its computation in insulators is no more necessary because the magnetic field constant value is equal by continuity to the one on the boundary of the contiguous conductor. As a consequence, the outer field in open boundary problems does not need to be solved.

The total current crossing a conductor can be imposed by Dirichlet boundary conditions. Indeed, the application of the Ampere theorem to a particular rectangular contour including any points P_1 and P_2 in the cross section enables to write a relation between the magnetic fields at those points. This contour is composed on the one hand of two one-meter-length parallel segments beginning at P_1 and P_2, parallel to z axis, and on the other hand of two segments parallel to the section. The following relation can then be obtained:

$$h(P_1) - h(P_2) = I \tag{5}$$

where I is the current per unit of depth (A/m) flowing between P_1 and P_2. This relation extended to the boundaries of the conductor enables to impose the magnetic field gap between them (Fig. 1).

The boundary conditions expressed in terms of the current density can be transformed into conditions on the magnetic field, i.e.

$$j_n = \mathbf{j} \cdot \mathbf{n} = \text{curl } \mathbf{h} \cdot \mathbf{n} = \frac{\partial h}{\partial s} , \quad j_t = \mathbf{n} \times \mathbf{j} = \mathbf{n} \times \text{curl } \mathbf{h} = \frac{\partial h}{\partial n} \mathbf{e}_z , \tag{6-7}$$

where s is the curvilinear coordinate along the boundary and **n** the normal vector.

Thus, a boundary next to an insulator is characterised by a constant magnetic field (Dirichlet condition); a boundary with normal current density (entering or going out) is characterised by a null normal derivative of the magnetic field (homogeneous Neumann condition).

Finally, the problem could be posed as illustrated in Fig.1.

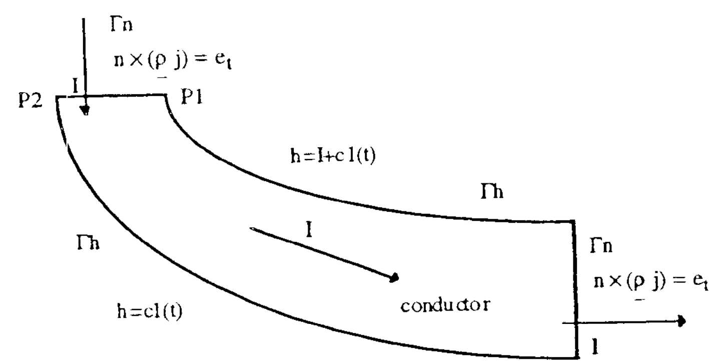

Fig. 1. A cross section of a conductor with a parallel current.

The choice of the spatial constant c1(t) in Fig.1 is important because it influences the transient solution and in particular the location of the skin effect. A physical analysis can help to find this constant. Suppose that initial solution is zero everywhere. If the current has to appear similarly on both conductor sides, the constant must be -I/2; if skin effect is expected to appear at first on the upper conductor side due to symmetry reason, a zero constant is taken. This will be the case for the considered application.

3. Finite element equations

The *finite element method* is implemented using nodal elements (triangles and quadrangles) which can be considered as prismatic or cubic edge elements with one meter length edges oriented along z axis. The nodal degrees of freedom are the magnetic field vectors along z axis. By combining the Faraday law, the Ampere law, the local Ohm law and the magnetic law, the continuous problem to solve (8) is :

$$\text{curl}\ (\rho\ \text{curl}\ h) + \partial_t(\mu(h)\ h) = 0 \quad \text{in} \quad \Omega \tag{8}$$

where Ω is the studied domain. Dirichlet and Neumann boundary conditions are imposed on Γ_h and Γ_n (Fig.1) taking into account the magnetic field gap due to the total crossing current.

The finite element equations are constructed by using the *weighted residual method* for (8) and the Neumann condition. The *Galerkine method* is applied, in which the test functions are the same as the shape functions. The discretised form of the magnetic field and of a finite element equation can be written as follows:

$$h = \sum_{j=1}^{Nn} h_j\ \alpha_j\ , \tag{9}$$

$$\sum_{j=1}^{Nn} \left\{ \int_{\Omega} (\ \text{rot}\ \alpha_i\ \rho\ \text{rot}\ \alpha_j\ d\Omega \right\} h_j + \sum_{j=1}^{Nn} \partial_t \left\{ \int_{\Omega} \mu\ \alpha_i\ \alpha_j\ d\Omega\ h_j \right\} + \int_{\Gamma_n} e_t\ \alpha_i\ d\Gamma_n = 0 \tag{10}$$

$$i = 1,\dots,Nn$$

where α_i are the test functions, α_j are the shape functions, h_j are the degrees of freedom, e_t is the imposed tangential value of the electrical field $(\mathbf{n} \times \mathbf{e})$ on the Neumann boundary and Nn is the number of nodes in the finite element mesh.

Since the magnetic permeability depends on the magnetic field, it also depends indirectly on time and therefore, it must remain under the time derivative in equation (10). This term introduces non-linearity in the equation. The problem is solved by using the Newton-Raphson method with an under-relaxation factor [3] which is small at the beginning of the iterative process and tends towards unity after some iterations. This choice enables the convergence in the case of strong non-linearity.

4. Application

The application concerns fuselage panel connections of an aeronautic axisymmetrical structure. Due to the geometry, a panel junction can be considered as a 2D plane problem because the thickness is small compared with the mean fuselage radius. When this structure is impacted by a lightning pulse (Fig. 2), a current density appears at first on the outer side. Indeed, since there is no current inside the fuselage (air), the Ampere law ensures that the magnetic field is always equal to zero on the inner skin. Thus, a null constant is taken for Dirichlet condition and the magnetic field on the outer boundary is equal to the total current per unit of depth. Homogeneous Neumann condition is fixed for the entry (left) and exit (right) sections in order to make the current lines (iso h) perpendicular to them; it physically means that the panels are long compared to junction dimensions and that the current continues straight in the panel direction.

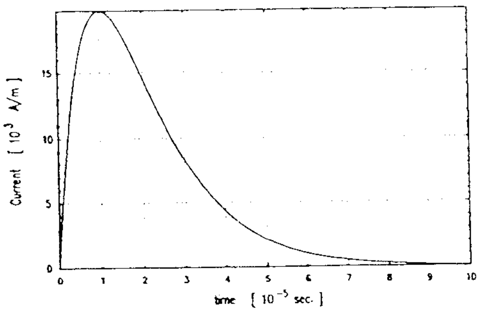

Fig. 2. Current wave.

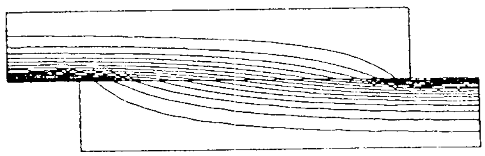

Fig. 3a. Current lines at time $t = 2\ 10^{-5}$ s.

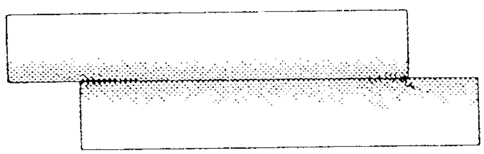

Fig. 3b. Joule losses at time $t = 2\ 10^{-5}$ s.

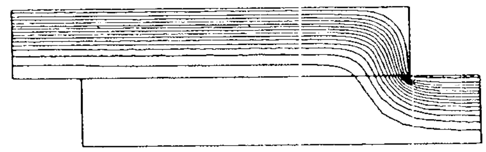

Fig. 4a. Current lines at time $t = 2\ 10^{-5}$ s.

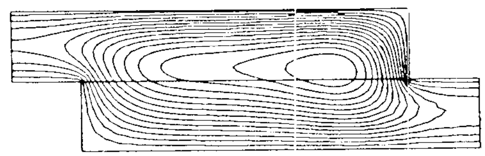

Fig. 4b. Current lines at time $t = 1\ 10^{-4}$ s.

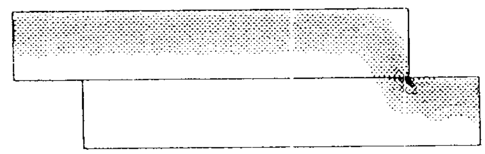

Fig. 4c. Joule losses at time $t = 2\ 10^{-5}$ s.

Fig. 3a and 3b present results for a carbon epoxy anisotropic material panel junction. Fig. 4a, 4b and 4c give results for aluminium material. Skin depth phenomenon is not perceptible for carbon epoxy due to the large vertical resistivity of the carbon epoxy. The anisotropic resistivity (lower in the horizontal direction than in the vertical direction) also explains the Joule losses location at the panels interface, where the current density has the greatest vertical component. Fig. 4b points out the presence of a current loop (current going back to the left on the upper side and going to the right on the lower side). Parallel current lines flowing in opposite directions are known to repulse each other (Laplace force). Therefore, dislocation between metallic panels may occur when the current is sufficiently concentrated.

5. Conclusions

A 2D magnetic field formulation has been developed and applied to aeronautic structures. Anisotropic resistivity tensor has been introduced into the local Ohm law to treat particular composite materials. The way to impose current constraints has been detailed. The application to aluminium or carbon epoxy panel junction has been performed. The heat losses distribution and the dislocation phenomenon in metallic structures have been pointed out.

References

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[3] Fujiwara K., Nakata T., Okamoto N., Muramatsu K., "Method for determining relaxation factor for modified Newton-Raphson method", IEEE Transactions on Magnetics, Vol.29, No.2, pp. 1962-1965, 1993.