

Modelling and finite element simulation of non-linear and anisotropic quasi-static electromagnetic systems

Author: Vande Sande Hans

Promotors: Hameyer K. & Henneberger G. (RWTH Aachen)

K.U.Leuven, ESAT-ELECTA

Abstract

This work contributes to the research on numerical simulation of electric energy transducers. The finite element method is used for modelling and simulating static and time-harmonic boundary value problems. Due to the ferromagnetic properties of electrical steel, this requires the numerical solution of a system of non-linear equations. The strong interplay between magnetic material modelling and non-linear simulation techniques is the basis of this work.

First, it is discussed in which way the non-linear and anisotropic behaviour of electrical steel can be modelled, starting from unidirectional measurement data. This model is used to develop a lossless time-harmonic anisotropy model. It is shown in which way losses can be incorporated in a time-harmonic anisotropy model. The system equations are derived for both, the static and time-harmonic problem. Two iterative methods for solving non-linear problems are presented. Exact and modified expressions for the Jacobian are derived, yielding a set of Newton, modified Newton and Picard iterative solvers. All previous aspects are thoroughly discussed by means of three technically relevant examples: the simulation of a switched reluctance motor, the no-load operation of a three-phase transformer and the standstill operation of an induction machine. At the end, two solver acceleration techniques are discussed.

Detailed overview of the work

The behaviour of many physical systems can be described by one or more partial differential equations. For most technically relevant problems, the solution of these equations cannot be determined analytically. Nowadays, numerical simulation techniques provide an outstanding alternative, when compared to traditional methods for solving such problems. The application area is wider and the amount of assumptions to be made is reduced considerably. In this work, the finite element method (FEM) is used to determine the numerical solution of quasi-static electromagnetic boundary value problems. Other methods, such as the boundary element method (BEM), the finite difference method (FDM) or the finite integration technique (FIT) are not considered here.

Basically, the finite element method is used to transform a non-linear partial differential equation into a system of non-linear algebraic equations, taking into account boundary conditions. The actual structure of the equation system depends on

the type of problem to be solved. The non-linearity of the equations, due to the use of non-linear and anisotropic materials, implies the use of iterative solvers. The strong interplay between material modelling and non-linear iterative techniques is the basis of this work.

Chapter 2, *Electrical Steel*, proposes a novel method for determining the reluctivity tensor entries from unidirectional measurements of the magnetisation curves of grain-oriented silicon steel exhibiting Goss-texture. Due to the anisotropy of this material, the applied field strength and the resulting flux density are generally not in parallel. Unidirectional measurements do not provide enough data to estimate the angle between these quantities. However, this angle is required for modelling the anisotropic behaviour correctly. The missing information is retrieved from a physical model for the anisotropy of cubic iron crystals. The crystal orientation distribution function, being the link between the microscopic and macroscopic level, is involved in the analysis as well. Finally, one ends up with the reluctivity tensor entries in the rolling and transverse direction, as a function of the magnitude and the direction of the flux density.

Chapter 3, *Finite Element Modelling of Quasi-Static Systems*, gives an overview of the equations that might occur in quasi-static electromagnetic field problems. The weak form of static and time-harmonic boundary value problems is used to determine the structure of the corresponding systems of non-linear equations. Much attention is paid to the tensorial representation of the reluctivity. For the magnetostatic case, it is discussed in which way orthogonal polynomials can be used for that purpose. For the time-harmonic case, the concept of effective magnetisation curves is generalised, to allow for non-linear anisotropic materials as well. Moreover, the reluctivity tensor entries may be complex-valued, to incorporate losses in the material model itself. Both considerations yield a time-harmonic complex-valued reluctivity tensor, whose entries depend on the three parameters defining the geometrical properties of the flux density locus. Perceptron neural networks are used to model the dependences.

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Chapter 4, *Solving Quasi-Static Finite Element Problems*, provides a mathematical platform for solving non-linear problems in general. By introducing the quadratic model concept, it is shown that a so-called trust region method can be used for solving non-linear magnetic problems as well, next to the widespread line search method. Both methods have similar convergence properties. In the context of solving systems of non-linear equations, they require expressions for the Jacobian of the residual. The Jacobian is analytically determined for both, static and time-harmonic problems. In the time-harmonic case, this requires a doubling of the system size, because the Cauchy-Riemann equations are not fulfilled. Since the use of the exact Jacobian may not be appropriate, modified expressions for the Jacobian are suggested. It is discussed in which way these modifications influence the convergence rate.

Chapter 5, *Selected Examples*, highlights all the basic aspects of the previous chapters, by means of three technically relevant problems in electromagnetics: the switched reluctance motor, the no-load operation of a three-phase transformer and the standstill operation of an induction machine.

Chapter 6, *Solver Acceleration Techniques*, discusses two methods for reducing the overall computation time. The first acceleration method exploits the fact that, in the time-harmonic case, Picard steps are less expensive to compute than Newton steps. It is demonstrated that a significant improvement can be achieved by initiating the iteration process by the Picard method and by switching to the Newton method once the latter is expected to converge faster. In this chapter, an estimator for the switching

moment is developed. The estimator is based on the quadratic model concept. The second acceleration method is generally applicable and continuously modifies the relative linear system solver tolerance while iterating. An empirical adaptation algorithm, based on two efficiency indicators, is proposed. An example illustrates that convergence is obtained earlier than in the theoretically based case.

Chapter 7, *Conclusions*, collects conclusions and presents suggestions for further research.