Bifurcation Analysis of Three-Phase Ferroresonant Oscillations in Ungrounded Power Sytems

T.Van Craenenbroeck
Member, IEEE

W.Michiels
Student Member, IEEE

D.Van Dommelen
Senior Member, IEEE

K.Lust

Dept. of Electrical Engineering
K.U.Leuven
3001 Leuven, Belgium

I.M.A.
Univ.of Minnesota
Minneapolis

Abstract: The study of power networks prone to ferroresonant oscillations by means of simulation turns out to be a tedious task. Therefore, a direct calculation of the stability domains of the various oscillation modes is preferable. In this paper, the software package AUTO is used to determine the periodic solutions of the differential equations describing a typical three-phase network in which ferroresonance has occurred. A small modification of the system equations is introduced to approximate the quasi-periodic oscillations by periodic ones, without major impact on the position of the stability domains. Finally, an example of the automated dimensioning of a damping circuit using a continuation scheme is presented.

Keywords: Bifurcation, Ferroresonance, Voltage transformers

I. INTRODUCTION

Ferroresonance is due to the interaction between a nonlinear inductance and a capacitance. The phenomena are initiated by a switching operation or a disturbance such as a temporary short circuit. Depending on the initial conditions, several oscillation modes can exist for the same network parameters.

Both the network configurations in which these phenomena can occur, as well as the oscillation modes, have been the subject of various studies. An overview of possibly dangerous network configurations is given in [1]. In this study our attention will be focused on the three-phase ferroresonance occurring in networks with ungrounded neutral as represented on Fig.1. The use of the presented method however is by no way restricted to this type of ferroresonance, provided an accurate description of the system by means of differential equations is available.

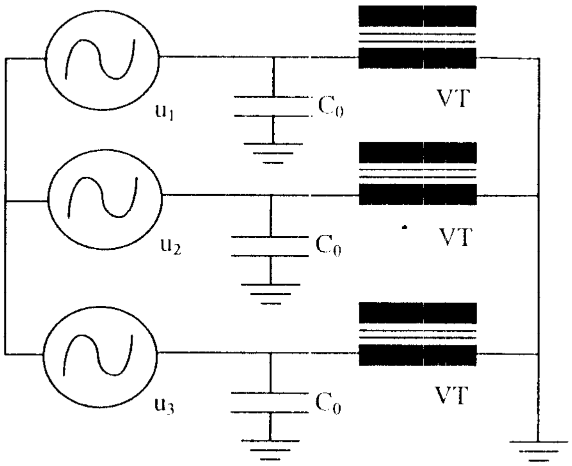

Fig.1. Network with ungrounded neutral

The network consists of a balanced three-phase voltage supply $\{u_1, u_2, u_3\}$ with ungrounded neutral, a feeding cable represented by its zero-sequence capacitance C_0 and three inductive voltage transformers VT. This configuration is used in power plant auxiliaries and distribution networks of large factories. Temporarily isolated public distribution networks can have the same configuration as well.

The ferroresonance phenomena in this circuit can be elegantly explained by applying a transform which allows the zero-sequence behavior to be studied. The nonlinear inductances introduce a coupling between the sequential circuits, which feeds energy to the zero-sequence circuit. If this transfer equals out the losses in the circuit, a permanent oscillation may be sustained [2].

The oscillation modes have been described in [3]. The predominant ones are listed in Table I with their main zero-sequence oscillation component (f_0 is the system frequency).

TABLE I
OSCILLATION MODES

Mode	Symbol	Main component
Unbalanced fundamental	UF	f_0
Harmonic-3	H3	$3.f_0$
Quasi-periodic 1/2	QP1/2	$\cong f_0/2$
Quasi-periodic 2	QP2	$\cong 2.f_0$

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With exception of the UF-mode, all these oscillations are balanced. The first two are periodical, whereas the remaining two are quasi-periodical. Beside the fact that the stability domains of these oscillations can be calculated directly (i.e. without an excessive number of simulations to be performed), the proposed method offers the supplementary advantage that the physical link between these modes can be explained, resulting in a better insight in the nature of the ferroresonance phenomena.

II. SYSTEM MODELLING AND EQUATIONS

A very simple transformer model is chosen to reduce the order of the system under study. It consists of a series resistance R_s (representing the losses in the primary winding of the VT), a shunt resistance R_p (representing the iron losses in the core of the VT) and a nonlinear flux-current characteristic. The calculation results mentioned in this paper were obtained using data from a 6.6kV/110V VT whose magnetization characteristic was approximated by a fifth-order polynomial using least-squares fitting:

$$i(\phi) = k_1.\phi + k_5.\phi^5 \tag{1}$$

The three-phase network represented in Fig.2 introduces a slight modification with respect to Fig.1: the 3 zero-sequence capacitances have been replaced by a single capacitance $C_{eq} = 3.C_0$ between neutral and earth. This reduces the order of the system without altering the waveforms at the terminals of the nonlinear elements [4]. For sake of clarity only one phase is represented in detail.

As the ferroresonant oscillations are essentially zero-sequence phenomena, they can be damped by inserting an appropriately designed damping circuit in the delta-connected tertiary windings of the VT's. The most simple method of damping, though not always effective because of limitations on thermal dissipation, is the use of a resistance R_d.

The network of Fig.2 can be described by a system of four ordinary differential equations (ODE), which can be written in a canonical form using the normalized Clarke transform:

$$\begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} = C^T.\begin{bmatrix} i_1(\phi_1) \\ i_2(\phi_2) \\ i_3(\phi_3) \end{bmatrix} \qquad \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix} = C.\begin{bmatrix} \phi_\alpha \\ \phi_\beta \\ \phi_0 \end{bmatrix} \tag{2}$$

$$\text{with } C = \frac{1}{\sqrt{3}}\begin{bmatrix} \sqrt{2} & 0 & 1 \\ -\sqrt{1/2} & \sqrt{3/2} & 1 \\ -\sqrt{1/2} & -\sqrt{3/2} & 1 \end{bmatrix} \tag{3}$$

Applying the same transform to the supply with U the rms line voltage, gives

$$\begin{aligned} u_\alpha &= U.\sin(2\pi f_0 t) \\ u_\beta &= -U.\cos(2\pi f_0 t) \end{aligned} \tag{4}$$

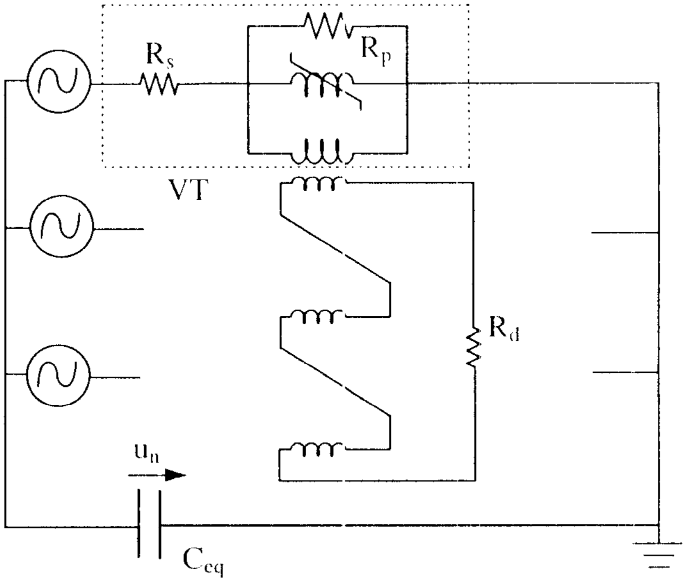

Fig.2. System representation

This results in the following system of ODE

$$\begin{aligned} \frac{d\phi_\alpha}{dt} &= \frac{R_p}{R_p + R_s}.(u_\alpha - R_s.i_\alpha) \\ \frac{d\phi_\beta}{dt} &= \frac{R_p}{R_p + R_s}.(u_\beta - R_s.i_\beta) \\ \frac{d\phi_0}{dt} &= \frac{R'_p}{R'_p + R_s}.(-\sqrt{3}.u_n - R_s.i_0) \\ \frac{du_n}{dt} &= \frac{-1}{C_0.(R_s + R'_p)}(u_n - R'_p.i_0/\sqrt{3}) \end{aligned} \tag{5}$$

$$\text{with } \frac{1}{R'_p} = \frac{1}{R_p} + \frac{3}{R_d} \tag{6}$$

The circuit parameters used in this study are:

$k_1 = 71.8E-6\ A/Wb$ $\qquad R_s = 700\,\Omega$

$k_5 = 2.58E-9\ A/Wb^5$ $\qquad R_p = 2M\Omega$

Initially no damping is assumed ($\frac{1}{R_d} = 0$). In section VI the influence of the damping resistance will be studied. The vector of unknowns is represented by

$$X = [\phi_\alpha \quad \phi_\beta \quad \phi_0 \quad u_n]^T \tag{7}$$

III. BIFURCATION THEORY

Bifurcation theory has proved to be the adequate mathematical framework for the study of nonlinear dynamical systems. An introduction to the principles of this theory as well as a case study (one-phase ferroresonance in a transmission system) can be found in [5].

The application of the bifurcation theory implies the calculation of a solution of the system of ODE with respect to a parameter λ. The critical values of this parameter where the type or the number of solutions of the system changes, are called the bifurcation values.

Different approaches exist to solve the system, amongst them the Galerkin method, which transforms the problem into the frequency domain [6]. In this paper the software package AUTO [7] was used. This freely distributed package (via ftp://ftp.cs.concordia.ca/pub/doedel/auto) has been widely used in several engineering disciplines. So far however, the number of practical applications in electrical engineering is limited. It's the authors' strong belief that ferroresonance case studies that are expressed as a system of differential equations, can be elegantly tackled within the framework of the AUTO package. It holds three immediate advantages over time domain studies:

1. Steady state solutions are directly computed.
2. The existence of multiple solutions, that may otherwise have been overlooked, can be predicted.
3. The stability domains of the solutions can automatically be determined.

AUTO computes and continuates the solutions of systems of algebraic and autonomous differential equations. The equations are discretized by an orthogonal collocation method [8]. The computation of periodical solutions can be treated as a boundary value problem.

A periodically forced system of order n can be transformed into an autonomous sytem by adding a stable nonlinear oscillator with the desired pulsation $\omega = 2\pi f_0$:

$$\begin{aligned} \dot{u}_\alpha &= \omega . u_\beta + (U^2 - (u_\alpha^2 + u_\beta^2)) \\ \dot{u}_\beta &= -\omega . u_\alpha + (U^2 - (u_\alpha^2 + u_\beta^2)) \end{aligned} \tag{8}$$

The solution of these equations corresponds to the forcing terms of (5), which are given in (4). By coupling (8) to the system (5), a sixth-order autonomous system is formed that can be solved by AUTO.

The software uses a pseudo-arclength continuation strategy to compute a branch of solutions. The new solution (X_1, λ_1) will be sought on a hyperplane perpendicular to the tangent determined by $(\dot{X}_0, \dot{\lambda}_0)$, on a prescribed distance Δs from the previous solution (X_0, λ_0).

In order to have variables of comparable magnitude, scaling of the voltages has been applied, expressing them in kV. The computed 1-parameter branches will be presented using the L2-norm as a characteristic measure of the obtained T periodic solution:

$$\|X\| = \sqrt{\int_0^T \frac{1}{T} \sum_{i=1}^{4} x_i^2(t) dt} \tag{9}$$

Stable solutions will be distinguished from unstable ones using dots and circles respectively.

IV. BIFURCATION ANALYSIS OF PERIODIC OSCILLATIONS

A. Balanced Oscillation Mode

Starting the continuation from no-voltage conditions, the behavior of the transformer cores is linear. This implies a normal, balanced three-phase operation with a negligible harmonic content. At a supply voltage of 14.6kV a torus bifurcation (TR) is detected (Fig.3). From this point onward the solutions on the branch are unstable, indicating the loss of normal behavior. At the TR a new branch of solutions originates, which are no longer periodical but quasi-periodical. The study of these solutions will be dealt with in section IV.B.

The branch of unstable solutions continues past a first limit point (LP) and a second TR, and regains stability after a second LP. Meanwhile the harmonic content of the phase fluxes and the voltage u_n has increased -especially the 150Hz component- indicating the transition to a H3 type of oscillation. The existence of LP involves the necessity of branch continuation above U_{nom}.

It is clear from Fig.3 that a region of supply voltages exists where both the normal and the ferroresonant oscillation can occur. In such cases the steady state oscillation mode will be determined by the initial conditions.

Finally, the H3 oscillation in turn loses stability at a new TR. The oscillations that originate at this point will not be studied in detail because their practical importance is very limited.

The question arises how these phenomena depend on C_0. As AUTO has the possibility of freeing a second parameter, the bifurcation point in a 2-parameter space may be determined. This can be applied to the continuation of both LP and TR.

In Fig.4 the results of the continuations of the LP are shown (dashes for lower LP, full line for upper LP), combined with 3 continuations on U. This 3D bifurcation diagram shows a surface with a fold that disappears at a so-called cusp point. For C_0-values below 1.7nF the continuation shows no LP, which means that there is a continuous transition from normal behavior to ferroresonance with main oscillation at 150Hz.

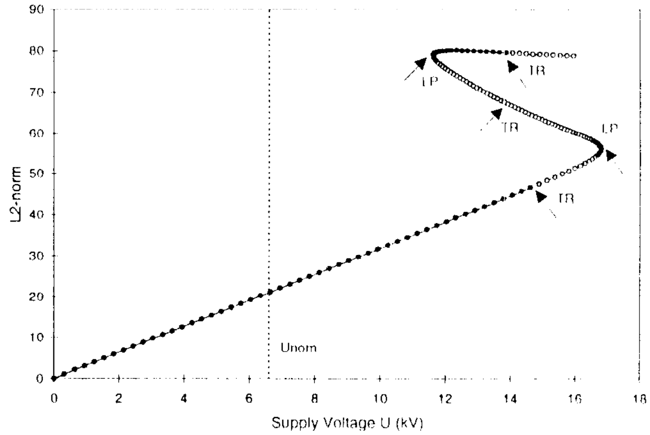

Fig.3. Continuation of a balanced oscillation at C_0 = 40nF

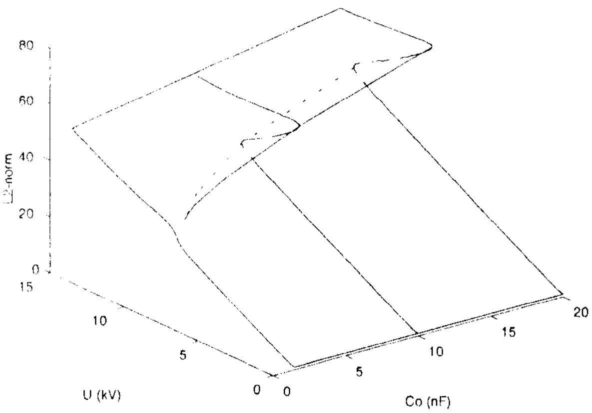

Fig.4. Balanced oscillations cusp

B. Unbalanced Oscillation Mode

The study of the Unbalanced Fundamental (UF) mode with AUTO requires an analytical starting solution, as there is apparently no continuous transition from balanced to unbalanced operation. This solution can be obtained by numerically integrating the system of ODE for a given value of the supply voltage. As the waveforms of the UF mode are quite similar to those of a line-ground fault [9], convergence of the integration routine to a steady state UF mode is facilitated with a high initial value of the zero-sequence flux and the neutral voltage.

The continuation of this UF mode at C_0=33nF is shown on Fig.5. The bifurcation diagram has the shape of a closed loop. A similar branch of solutions is called an isola. For every value of the supply voltage a stable and an unstable UF oscillation exist, joining each other at the LP at both sides of the branch. No other bifurcation points were detected during the computation. As there is no preferential phase and the behavior of the three phases is totally different (one or two phases can be heavily saturated, whereas the other phase(s) is/are nearly unloaded), two similar UF modes exist by simply shifting the phases. The solution branches of these modes coincide with the one shown on Fig.5 because the L2-norm uses information of all phases.

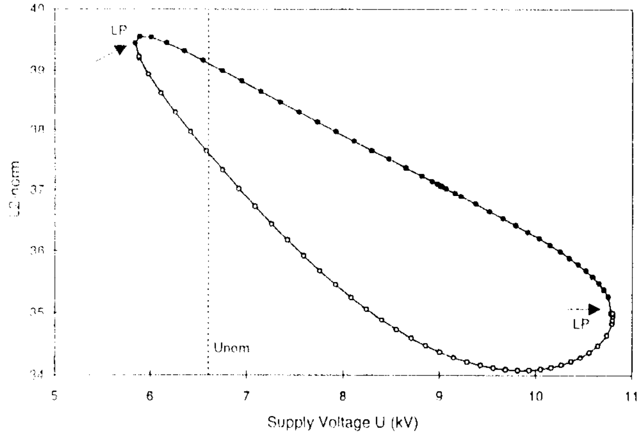

Fig.5. Continuation of UF mode at C_0=33nF

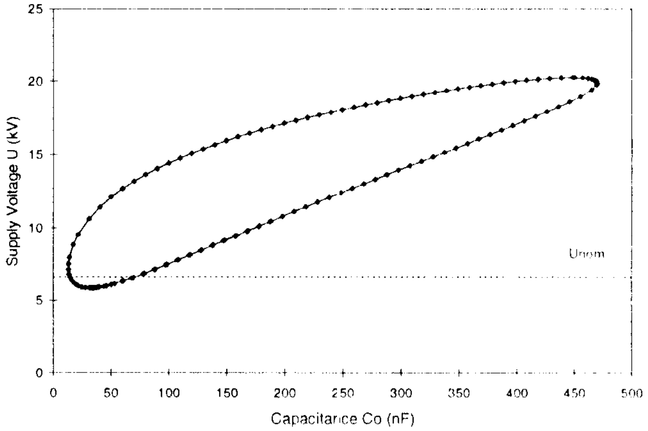

Fig.6. Stability domain of UF mode

The continuation of the LP of Fig.5 with respect to U and C_0 determines the stability domain of these oscillations (Fig.6). It's important to notice that even below nominal supply voltage (6.6kV) UF oscillations can be sustained for a region of (realistic) C_0-values around 50nF. The jump from balanced to unbalanced operation can be initiated by any kind of disturbance, but there is no continuous transition. As the shift of the neutral point exceeds the neutral voltage at a one-phase to earth fault, this oscillation mode has to be avoided by all means.

V. BIFURCATION ANALYSIS OF QUASI-PERIODIC OSCILLATIONS

A. Problem formulation

Although AUTO is able to detect the TR, continuation of the emanating branches of quasi-periodic oscillations is impossible. This impedes the prediction of the lowest supply voltage where those oscillations can be sustained. The ignorance is even complete in the case of isolas of quasi-periodic behavior, for these gain and lose stability at LP, which can never be reached by continuation.

The question arises whether the quasi-periodic oscillations can be approximated by periodic ones in order to be continued. Any proposed approximation scheme must not only be evaluated by inspection of the waveforms, but must necessarily compare the stability domains of the (exact) quasi-periodic and (approximated) periodic oscillations.

B. System approximation

If the series resistance R_s is neglected in the equation describing the β-circuit (or equivalently in the α-circuit equation), the flux variation is directly linked to the sinusoidal voltage supply [2]. In this way ϕ_β is fixed beforehand, assuming an initial value for this variable. By substituting the analytic expression of ϕ_β

$$\phi_\beta = -\frac{U}{\omega}.\sin(\omega t) \qquad (10)$$

the system is reduced to three ODE.

The proposed approximation implies the existence of a double periodic solution for every quasi-periodic oscillation. This fact can be explained by the observation that the slow frequency component in the waveforms of both ϕ_α and ϕ_β is eliminated by the approximation. Therefore, and with the previously chosen initial value $\phi_\beta = 0$, the computed periodic solution corresponds to the quasi-periodic waveform around the moment the slow component of ϕ_β changes sign. This happens twice a period when the slow component of ϕ_α is at its crest value (maximum or minimum). Both solutions coincide in the L2-norm representation.

C. Quasi-periodic 1/2

The QP1/2 mode turns out to be a balanced oscillation, driving each phase in a cyclic way into saturation. However, a continuous transition starting from the normal oscillation has not been detected, in spite of numerous continuations. Therefore an initial solution must be found by numerical simulation of the system of ODE. The bifurcation diagram is again an isola with two LP (Fig.7), as for the UF mode. The upper half of the solution branch exhibits two period-doubling bifurcations (PDB). These bifurcations are characterized by the fact that the solutions on the initial branch become unstable, while a new solution branch emanates with an oscillation having a double period. As the position of this new branch is close to the initial branch, it hasn't been added to Fig.7.

The continuation of the LP of Fig.7 is of utmost importance as this oscillation mode is the most likely to appear in a power system with ungrounded neutral. The lower voltage limit is well below nominal voltage, and the stability domain is stretched over a broad C_0-region. The computed stability domain is represented on Fig.8. To verify the errors introduced by approximating the system, stability limits obtained by simulation of the exact system have been added to Fig.8.

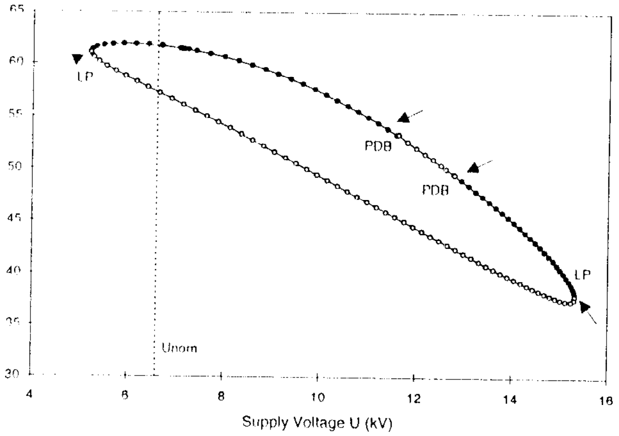

Fig 7. Continuation of QP1/2 mode at C_0=550nF

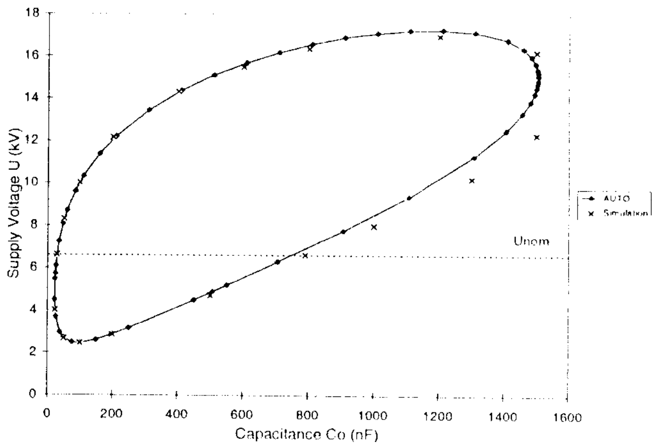

Fig.8. Stability domain of QP1/2 mode

D. Quasi-periodic 2

The same approximation can be applied to the QP2 mode, which is again a balanced quasi-periodic oscillation mode. Contrary to the previous mode, a direct link exists with the normal oscillation. Indeed, the TR detected in Fig.3 gives rise to the QP2 mode. This can be confirmed by repeating the continuation process for the approximated system (Fig.9), in which the TR is converted into a pitchfork bifurcation (PB). When the stability of the solutions on the first branch is lost, two solution branches emanate which, for reasons explained before, coincide on Fig.9. The property of transforming a TR into a PB cannot be overestimated, as the emanating branch now consists of periodic solutions which in turn can be continued. Only after a first LP the solutions on this branch become stable. A harmonic analysis confirms the fact that the solutions are indeed periodic approximations of the QP2 mode. As in the previous case PDB can be detected, with the appearance of components of half the system frequency in the stable oscillation, while the pure QP2 mode becomes unstable. For a short region between a second PDB and a second LP, the QP2 mode regains stability. Finally, the two coinciding branches join the main branch in a second PB.

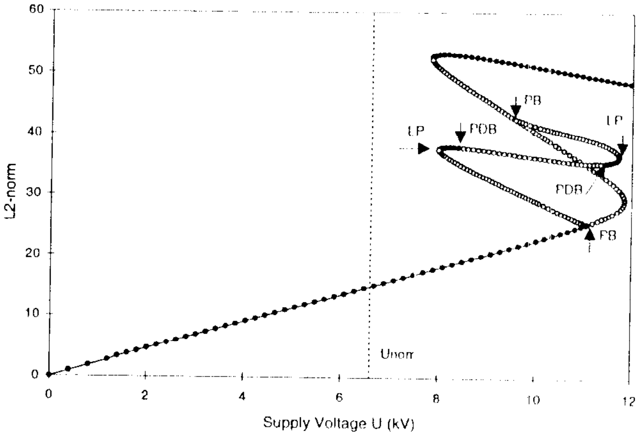

Fig.9. Continuation of QP2 mode at C_0=10nF

VI. DAMPING OF FERRORESONANT OSCILLATIONS

For as long as ferroresonance has been observed, power system engineers have found solutions to protect the system. Both active and passive measures have been considered [10]. Up till now however, no accurate computation eliminating any risk was possible.

The continuation technique can be used to automate the dimensioning of a damping circuit. The simplest solution is the use of a damping resistance in the Δ-connected tertiary windings. Fig.10 shows the results of the continuation of the QP1/2 mode for various resistance values. The crossing from one curve to another can easily be done using the resistance value as continuation parameter. In this way the stability domain of ferroresonant modes can be shifted toward higher supply voltages. For the VT under study, with UF and QP1/2 modes occurring below nominal voltage without damping resistance, the continuation shows that the UF mode is very sensitive to damping, while the elimination of QP1/2 oscillations needs a much smaller tertiary resistance.

VII. CONCLUSIONS AND FUTURE WORK

In this paper the use of continuation techniques to determine ferroresonant oscillations was presented and strongly recommended. The main advantages of this method are

- the improvement of physical insight in the nature of the oscillations.
- the automated computation of the stability domains of the oscillation modes.
- the automated dimensioning of efficient protective circuits.

In the near future this work will be extended to the study of nonlinear protective devices, that can have advantages in those cases where the thermal dissipation of the damping resistance becomes inadmissible. Besides more attention will be spent to the accurate modelisation of the voltage transformer. Finally, other approximation schemes for quasi-periodic oscillations will be evaluated.

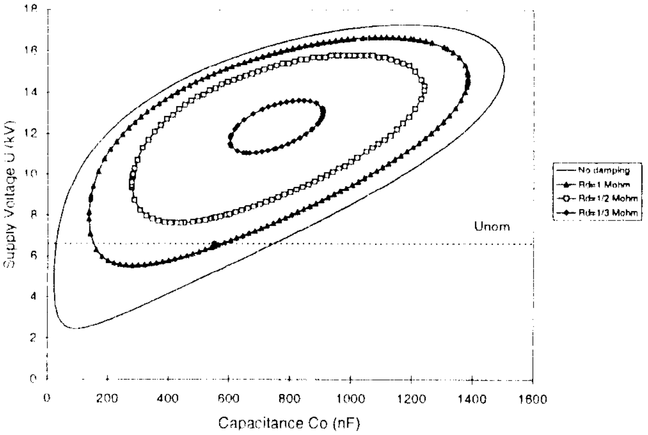

Fig.10. Damping of QP1/2 mode

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IX. BIOGRAPHIES

Thierry Van Craenenbroeck (M '96) graduated in 1989 as Electrotechnical Engineer from the Katholieke Universiteit Leuven (KULeuven). From 1990 to 1992 he worked as lecturer at the Anton-de-Kom University of Surinam. From 1992 on, he is working towards the Ph.D. degree in Electrical Engineering at KULeuven. His research topic is three-phase ferroresonance in distribution networks.

Wim Michiels graduated in 1997 at the same University. For his thesis, he studied the application of collocation methods in ungrounded circuits liable to ferroresonance. Particularly he investigated the possibilities of the software package AUTO with respect to the problem as treated in this paper.

Daniel Van Dommelen (SM '78) is Electrotechnical Engineer from the K.U.Leuven in Belgium, has an M.Sc. in Electrical Engineering (U. Wisc.), and a PhD from the K.U.Leuven. Since 1977 he is full professor at this university and head of the laboratory for Power Systems, High Voltage and Electroheat. He is teaching both undergraduate and graduate courses in these fields. He is author of a book on Production, Transmission and Distribution of Electric Power, presently in its fourth reprint, and author of numerous publications in both national and international journals. He is Belgian representative in the Education and Research Committee of the UIE. He has been chairman of the IEEE Benelux Section, and is a member of IEEE PES, CIGRE, SEE and national electrical engineering societies.

Kurt Lust graduated in 1992 as Engineer in Computer Sciences and obtained a Ph.D. in 1997 from the K.U.Leuven. He joined the IMA (U.Minnesota) for one year in September 1997. His main research interest is the development of efficient algorithms for the computation and bifurcation analysis of branches of periodic solutions of large-scale models with low-dimensional dynamics.