COMPARISON OF THREE METHODS TO STUDY FERRORESONANCE IN VOLTAGE TRANSFORMERS

T.Van Craenenbroeck D.Van Dommelen R.Belmans
Dept. of Electrical Engineering
K.U.LEUVEN
3001 Leuven, Belgium

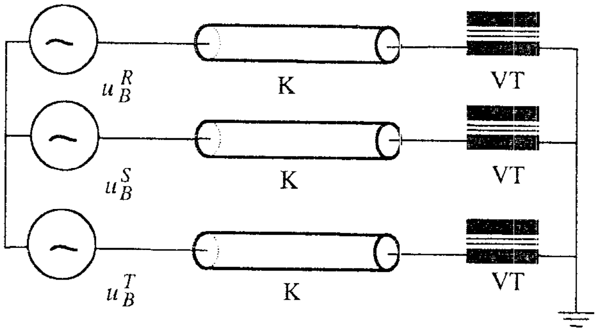
Abstract - The determination of ferroresonant oscillations in circuits with voltage transformers is a highly nonlinear problem due to the possible core saturation. Three different methods to study these oscillations and their stability domains are compared in this paper. The first method is based on a time domain numerical integration with simultaneous calculation of Floquet-multipliers. The second method uses orthogonal collocation to solve the system equations in the time domain. The third method is basically a frequency domain method, based on the harmonic balance principle, with special precautions taken for the nonlinear circuit elements. Advantages and drawbacks of these methods are first discussed for periodic oscillations. Then adapted methods for the study of quasi-periodic oscillations are presented. The methods can easily be extended to study the damping effect of a tertiary resistor. Results are shown for a typical configuration.

1. - Introduction

Ferroresonance is due to the interaction between a nonlinear inductance and a capacitance. The nonlinear inductance is typically the saturable magnetizing inductance of a transformer, whereas the capacitance can be ascribed to distribution cables, transmission lines, capacitor banks, voltage grading capacitors in HV circuit breakers or by the coupling between double circuit lines. The ferroresonant phenomena are initiated by a switching operation or a disturbance such as a temporary short circuit. Typically, several oscillation modes can exist for the same network elements. Most of these modes involve excessive transformer heating due to high peak currents and insulation failures due to overvoltages.

An overview of network configurations in which these phenomena can occur is given in [1]. In this study our attention will be focused on the three-phase ferroresonance occurring in networks with ungrounded neutral as represented on Fig.1. It consists of a balanced three-phase voltage supply $\{u_B^R, u_B^S, u_B^T\}$ with ungrounded neutral, a feeding cable K with capacitance C_0 to ground and three inductive voltage transformers VT. This configuration is used in power plant auxiliaries and distribution networks of large factories. Public distribution systems can be floating for short times as well.

Fig.1: *Network with ungrounded neutral*



In this configuration each transformer can be described separately, since there is no magnetic coupling between phases. The described methods, however, are applicable to other types of ferroresonance as well, provided an accurate description of the magnetic behavior of the transformer is available.

2. - Oscillation modes

The ferroresonance phenomena in this circuit can be described [2] by applying instantaneous Clarke-transform in order to study the zero-sequence behavior of the circuit. Using this technique the ferroresonance in Fig.1 can be explained as a resonant oscillation of the zero-sequence circuit, involving the capacitance to ground of the feeding cable and the inductance of the voltage transformer. Energy transfer from the α- and β-circuits to the zero-sequence circuit is needed to sustain the oscillation and is actually obtained due to the nonlinear elements. The main oscillation modes and their predominant components are summarized in Table 1, where f_0 represents the system frequency.

Table 1: *Oscillation modes*

Mode	Symbol	Main component
Unbalanced fundamental	UF	f_0
Harmonic-3	H3	$3.f_0$
Quasi-periodic-½	QP½	$\cong f_0/2$
Quasi-periodic-2	QP2	$\cong 2.f_0$

Unlike the periodic oscillation modes UF and H3, quasi-periodic oscillations cannot be described as a sum of integer harmonics of one base frequency. An exact description of these oscillatione would involve all linear combinations of two incommensurable base frequencies.

Subharmonic oscillations of order 2 and 3 have been reported in [3]. These oscillations are only stable for very limited parameter ranges. The proposed methods for periodic oscillations can be applied to these subharmonic oscillations as well, if the base period is correspondingly doubled or tripled.

The zero-sequence behavior of the four oscillation modes of Table 1 is represented on the phase portraits of Fig. 2a-d. The voltage at the system's neutral u_N and the zero-sequence component of the flux linkage ϕ_{NL}^0 are shown on Y- and X- axis respectively. The Poincaré-map (representation with one point per period of the source) and the direction on the trajectory are indicated.

The shape of the trajectories of UF (Fig.2a) and H3 (Fig.2b) looks very similar. The major difference is the fact that the UF-trajectory performs one loop per period, whereas the H3-trajectory consists of 3 coinciding loops per period of the fundamental frequency.

Fig.2a: *Zero-sequence phase portrait of UF*

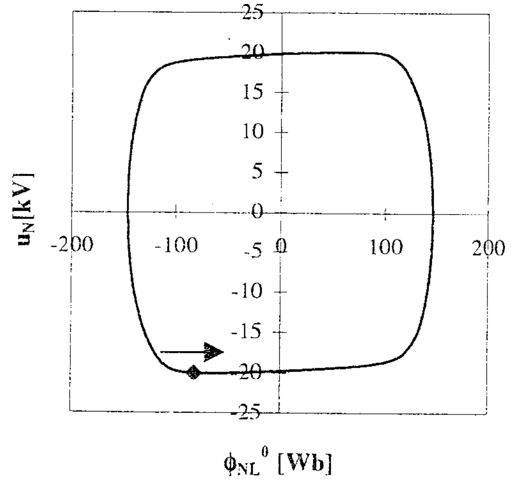


Fig.2b: *Zero-sequence phase portrait of H3*

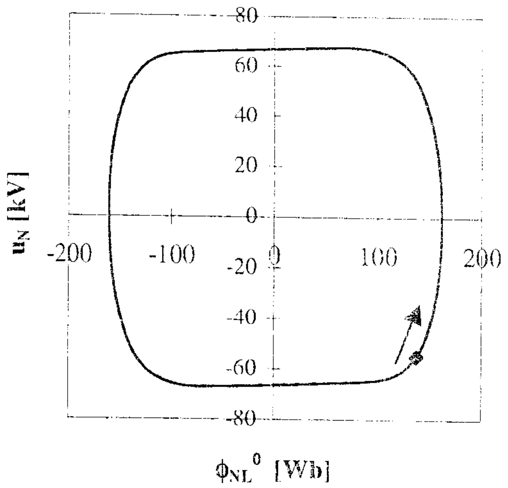


Fig.2c: *Zero-sequence phase portrait of QP½*

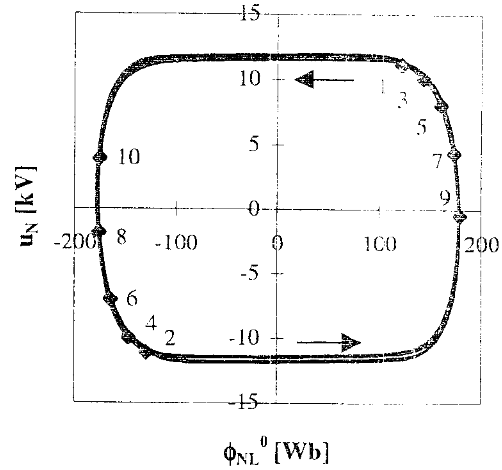


Fig.2d: *Zero-sequence phase portrait of QP2*

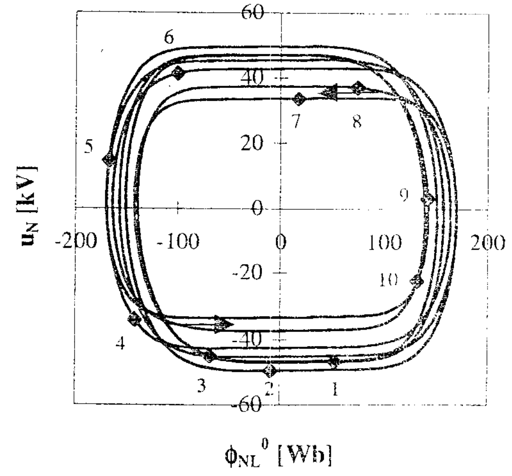


Fig.2c shows the evolution of the QP½-oscillation during 10 periods of the source. It can be seen that the state variables u_N and ϕ_{NL}^0 are almost periodic at a frequency slightly lower than $f_0/2$. The markers representing the Poincaré-map indicate this slip.

Fig.2d shows the evolution of the QP2-oscillation during 10 periods of the source. The trajectory now describes almost two loops per period. The slip is in this case higher than for the previous picture.

3. - Calculation methods for periodic oscillations

3.1 – Introduction

The next three paragraphs describe three methods which can be used, both to study the ferroresonant oscillations themselves, as well as to compute their stability domains. Theoretical derivations will be given and applied to a configuration with three $\frac{6.6}{\sqrt{3}}kV / \frac{110}{\sqrt{3}}V / \frac{110}{3}V$ voltage transformators (VT) that are connected to the ungrounded mains by means of a cable. The primary resistance R_p and inductance L_p of this VT are 700 Ω and 2.16 H respectively. The iron losses are modelled with a constant resistance R_{Fe} of 2 MΩ. The magnetic characteristic can be approximated by a fifth-order polynomial:

$$i_{NL}(\phi_{NL}) = \kappa_1.\phi_{NL} + \kappa_5.\phi_{NL}^5 \tag{1}$$

with

$\kappa_1 = 71.8\,E-6\,A/Wb$

$\kappa_5 = 2.58\,E-9\,A/Wb^5$

The equivalent circuit is shown on Fig.3. Only one phase is shown in detail.

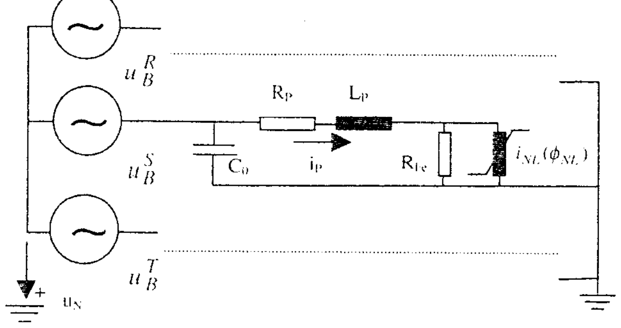


Fig.3: *Equivalent circuit*

The system equations can be expressed in the state variables $\left\{\phi_{NL}^{R.S.T}, i_p^{R.S.T}, u_N\right\}$:

$$\begin{aligned}
\frac{d\phi_{NL}^{R.S.T}}{dt} &= R_{Fe}.\left(i_p^{R.S.T} - i_{NL}^{R.S.T}\right) \\
\frac{di_P^R}{dt} &= \frac{u_B^{R.S.T} - R_{Fe}.\left(i_p^{R.S.T} - i_{NL}^{R.S.T}\right) - R_p.i_p^{R.S.T} - u_N}{L_p} \\
\frac{du_N}{dt} &= \frac{i_p^R + i_p^S + i_p^T}{3.C_0}
\end{aligned} \tag{2}$$

symbolically represented as

$$\frac{dx}{dt} = f(x,t) \tag{3}$$

The first and second method to solve this system of ODE are time domain methods, whereas the third method is a frequency domain method.

3.2 - Floquet-method

The classic time domain method consists in a transient simulation using a numerical integration program. A relevant network parameter λ (such as the capacitance) can be ramped upward and downward to detect qualitative changes in the solution [4], the so-called bifurcations. The parameter λ is called the bifurcation parameter.

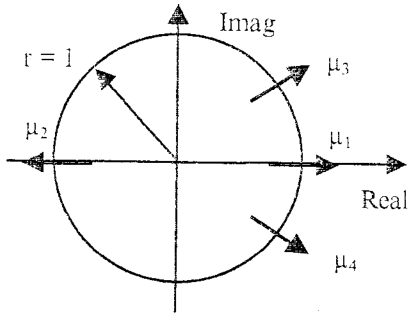
Bifurcations can be computed directly by simultaneously solving the system of variational equations [5, p.256]. This system of equations can easily be derived from the set of equations (2). The solutions of these variational equations enable us to investigate the stability of periodic solutions of (2) with period T. The variational system can be written under canonical form:

$$\begin{aligned}
\dot{\xi} &= Df(x_{eq}).\xi \\
\xi_0 &= I
\end{aligned} \tag{4}$$

with Df the Jacobian of the vector field f, I the identity matrix and x_{eq} the periodic solution.

The eigenvalues μ_i of the state transition matrix $\xi_T(x_{eq}) = e^{Df(x_{eq}).T}$ are called the characteristic multipliers or Floquet-multipliers. The stability of the solution is determined by the position of these multipliers in the complex plane. All the multipliers have to be inside the complex unit circle to prevent a small perturbation to grow with time. Change of stability occurs when one of the Floquet-multipliers passes through the unit circle with the variation of a network parameter. Three different configurations can be met (Fig.4):

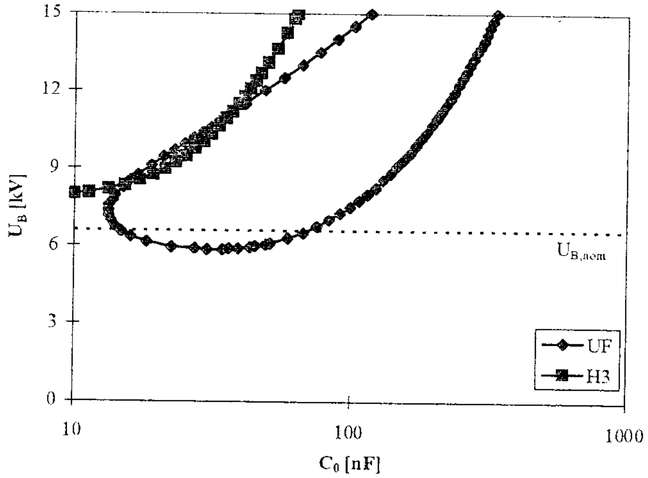
Fig.4: *Unit circle crossings of Floquet-multipliers*



- an eigenvalue μ_1 crossing the circle at the real value +1. Limit (or turning) points, transcritical and pitchfork bifurcations have this property.
- an eigenvalue μ_2 crossing the circle at the real value −1. This corresponds to a period doubling bifurcation. This implies that the original solution becomes unstable, and two new stable periodic solutions appear with doubling of the period.
- two complex-conjugate eigenvalues μ_3 and μ_4 crossing the unit circle. The bifurcation is then called bifurcation into a torus. This is the counterpart of a Hopf-bifurcation for periodic solutions [5, p.274]. At such a bifurcation point, the periodic solution becomes unstable and the new stable solution is quasi-periodic.

By freeing a second network parameter the stability domains can be traced. An algorithm has been implemented that controls the change in parameters in such a way that the time simulation tracks the stability limit of a periodic ferroresonant oscillation. The results for the UF- and H3-modes in the configuration under study, neglecting L_p, are represented on Fig.5.

Fig.5: *Stability domains of UF and H3 with Floquet-method*



This approach is straightforward but has some drawbacks. The variation of the network parameters inherently involves a transient with a decay time which is strongly dependent on the parameter margin with regard to the critical value at the bifurcation point. Consequently, an accurate detection of this critical parameter values imposes a very slow parameter change. Furthermore, a complete model including leakage inductances increases the stiffness of the system of differential equations drastically. Both preceding remarks stress the need for a time simulation with an extremely small time step, leading to a very long simulation time.

A second disadvantage is the fact that unstable solutions cannot be simulated. It can seem odd at first that this is a disadvantage, since unstable oscillations do not occur in the real world but only exist in the mathematical set of solutions. However, the gap between two stable oscillations can be bridged by a branch of unstable oscillations. This is sometimes seen as a jump in transient simulations, but might as well be overlooked.

3.3 – Orthogonal collocation method

A second time domain method disregards the transients and directly computes the steady state solutions of the system (3). The wellknown software package AUTO [6] uses an orthogonal collocation method to discretize the system equations and treats the computation of periodical solutions as a boundary value problem [7]. A periodically forced system can be transformed into an autonomous sytem by adding a stable oscillator with the desired pulsation $\omega = 2\pi f_0$ [8].

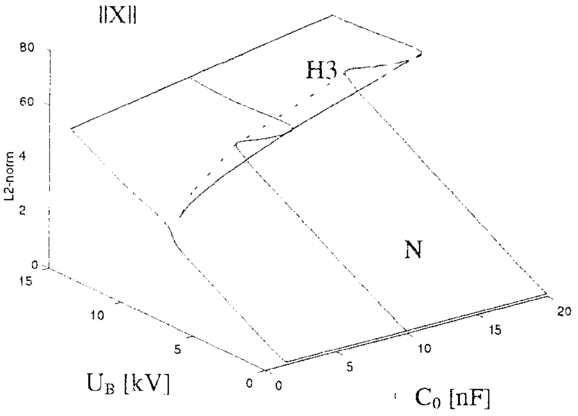
The software uses a pseudo-arclength continuation strategy to compute a branch of solutions. The new solution (X_1, λ_1) will be sought on a hyperplane perpendicular to the tangent determined by $(\dot{X}_0, \dot{\lambda}_0)$, on a prescribed distance Δs from the previous solution (X_0, λ_0) [9].

In order to have variables of comparable magnitude, scaling of the voltages has been applied, expressing them in kV. The L2-norm is used as a characteristic measure of the obtained T-periodic solution:

$$\|X\| = \sqrt{\int_0^T \frac{1}{T} \sum_{i=1}^{4} x_i^2(t)\,dt} \tag{5}$$

If the system solution is continuated with increasing system voltage for different values of C_0, starting from normal operation with the VT's working in the linear part of their characteristic, the bifurcation diagram of Fig.6 is obtained. It explains the transition from normal operation (N) to H3-ferroresonance. The solution surface is characterized by a fold that disappears at a so-called cusp point around $C_0 = 2$ nF and $U_B = 8$ kV. The solution between the two turning points on Fig.6 corresponds to the stability limit of the H3-oscillation as shown on Fig.5. Below $C_0 = 2$ nF a continuous transition from stable N to stable H3 can be observed. However, no transition exists to UF-ferroresonance. These UF-oscillations form an isola of solutions, as shown in [8].

Fig.6: *Transition to H3 with orthogonal collocation method*



Direct calculation of steady state solutions bypasses the problem of transients. This is a major improvement with respect to the Floquet-method. Moreover, branches of both stable as well as unstable solutions can be continuated, which makes it possible to explain the connection between different oscillation modes.

Major drawback of this method is the need to restrict the system order. Calculation time increases drastically with the dimensions and convergence can become problematic. Besides, the method requires fairly good initial conditions.

3.4 – Harmonic balance method

The system equations (3) can be transformed to the frequency domain, expressing the flux linkages of the nonlinear inductances by means of a truncated Fourier series:

$$\phi(t) = \Phi_0 + \sum_{k \in K} (\Phi_{k,c} \cos(k\omega t) + \Phi_{k,s} \sin(k\omega t)) \qquad (6)$$

where set K of the harmonic components is selected in accordance with the considered oscillation mode. However, the nonlinear characteristic (1) of the VT cannot be evaluated directly in the frequency domain. Therefore, the Fourier coefficients of the currents $I_k^{R,S,T}$ have to be calculated numerically, using the time evolution of the flux linkages.

The linear part of the circuit can be represented using the generalised Thévenin theorem at all the frequencies $k \in K$. In this way, the harmonic balance method can describe the system with one complex equation per nonlinear element for each harmonic component k.

$$jk\omega\Phi_k^{R,S,T} = U_{Th,k}^{R,S,T} - Z_{Th,k}^{R,S,T} \cdot I_k^{R,S,T} \qquad (7)$$

The efficiency of this method primarily depends on the number of harmonics that are needed to approximate the steady state with the desired accuracy [10, p.130]. Ferroresonant oscillations, especially the periodic ones, can adequately be described with a limited number of harmonics [11]. Consequently, the harmonic balance method is very well fit to study ferroresonance.

The system of nonlinear algebraic equations (7) can easily be solved using a general purpose Newton-Raphson scheme. In this way the problem is reduced to an algebraic bifurcation problem of the form

$$G(\Phi(k), \lambda) = 0 \qquad (8)$$

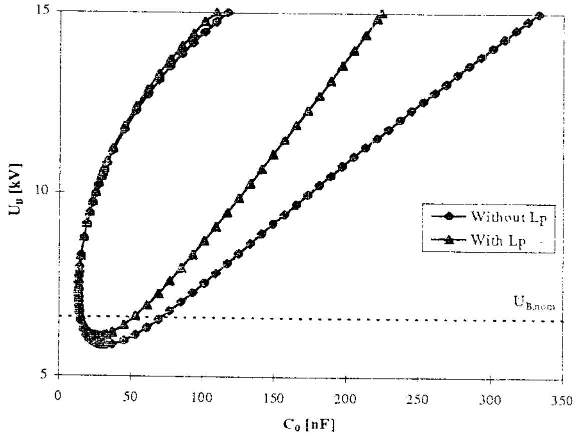
It is wellknown that the Jacobian J of this system becomes singular at limit (or turning) points and at transcritical and pitchfork bifurcation points [8]. Thus, the stability domains of the ferroresonant oscillations can be determined by adding the following equation

$$\det(J(\Phi(k), \lambda) = 0 \qquad (9)$$

and freeing a second parameter for the continuation.

The major advantage of this method is the ease with which the model of the linear circuit can be extended. In contrary to both preceding methods this doesn't increase the system order. In Fig. 7 the influence of the leakage inductance L_P on the stability domains of the UF ferroresonance is shown. For values of C_0 above 50 nF the calculation with neglect of L_p turns out to be a rough approximation. From a manufacturer's point of view, this approximation is on the safe side, but the design of a damping circuit can become problematic.

Fig.7: *Influence of L_p on stability domain of UP*



It's noteworthy that period doubling bifurcations (PDB) can be computed by including in (8) the equations and the flux linkage components relative to the harmonics which appear in the oscillation after the PDB. It's proven in [12] that a PDB of the system (7) corresponds to a pitchfork bifurcation of the extended system.

4. - Approximations for quasi-periodic oscillations

4.1 - Introduction

In order to apply the presented methods to determine the stability domains of quasi-periodic oscillation modes, approximations have to be introduced since periodicity was assumed in the derivations.

First an approximation to the time domain equations will be considered, leading to an extended use of the methods described in 3.2 and 3.3. Then the applicability of an approximation to the frequency domain equations is discussed. Features and results of both approximations will be compared in 4.4.

4.2 – Approximated time domain equations (ATD)

A time domain approximation has been presented in [2]. It consists in neglecting the series impedance in the β-circuit (or as an alternate in the α-circuit). It is proven that the low beat frequency is then shifted to a DC component, which makes the solution periodic and at the same time preserves a good correspondance with the real quasi-periodic oscillation. The approach could be regarded as a snapshot taken on the slow varying envelope of the oscillation.

The adapted system equations can be solved with both time domain methods. Relative advantages and drawbacks are similar as for periodic oscillations.

4.3 – Approximated frequency domain equations (AFD)

The frequency domain model can restrict the neglect of the series impedance to the DC cicuit equations. All harmonic equations can use the correct Thévenin-impedance at the frequency considered. This implies that the losses at the various frequency components are better represented in this model.

4.4 – Comparison

Stability domains of QP½ (Fig.8) and QP2 (Fig.9) with both approximations will now be compared, using very accurate simulation results as reference.

The upper border of the QP½ stability domain can very accurately be computed with both models. For the lower border, the agreement is good for low C_0-values but gets worse above 500 nF. The reason for this difference is the fact that the low frequency increases with C_0, thus the shift from f_L to DC involves a coarser approximation.

The results with the AFD-model are situated above the ATD-model results because of the higher losses in the AFD-model.

Fig.8: *Approximated stability domains of QP½*

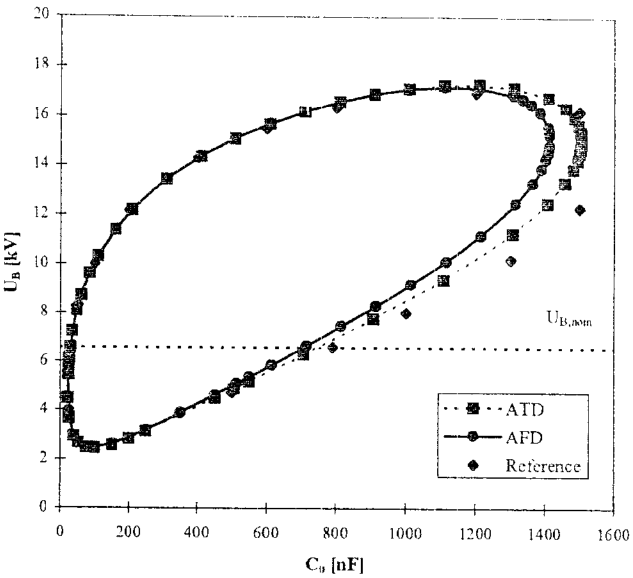
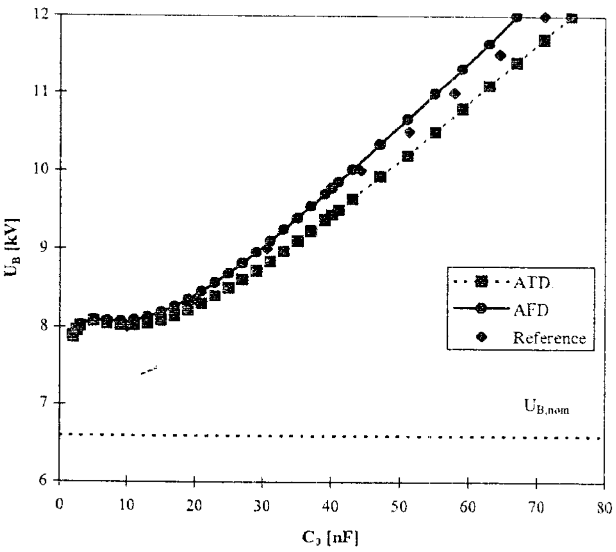


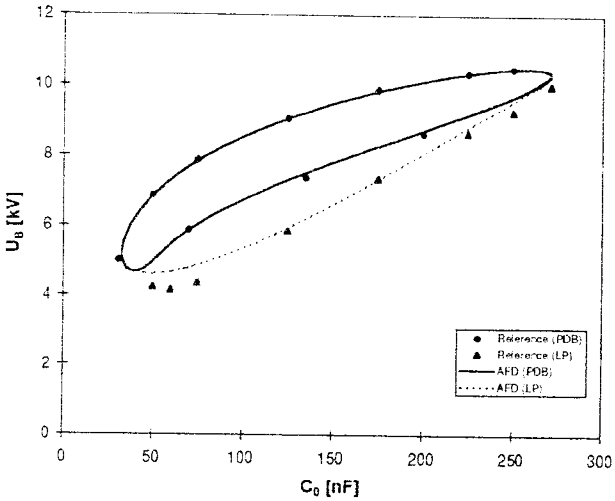
Fig.9: *Approximated stability domains of QP2*



Similar conclusions can be drawn for the QP2-oscillation. The difference with the accurate simulations again increases with f_L and thus with C_0. In contrary to Fig.8, the results from the two models are now situated on both sides of the reference solution, and accuracy for both models is about the same.

During simulations it was observed that components at about a quarter of the system frequency appear in the oscillation for certain parameter values within the QP½-stability domain. This phenomenon shows a hysteresis behaviour: the voltage where these components appear is different from the voltage where they disappear. With increasing voltage, the transition is a continuous period doubling bifurcation, whereas for decreasing voltages, the components suddenly vanish at a limit point. Both the methods can be used to determine these inner stability domains. However, the underestimation of the losses in the ATD-model causes a big difference when the results are compared with accurate simulations. The AFD-model on the other hand is very well suited to compute these domains, as is shown on Fig.10.

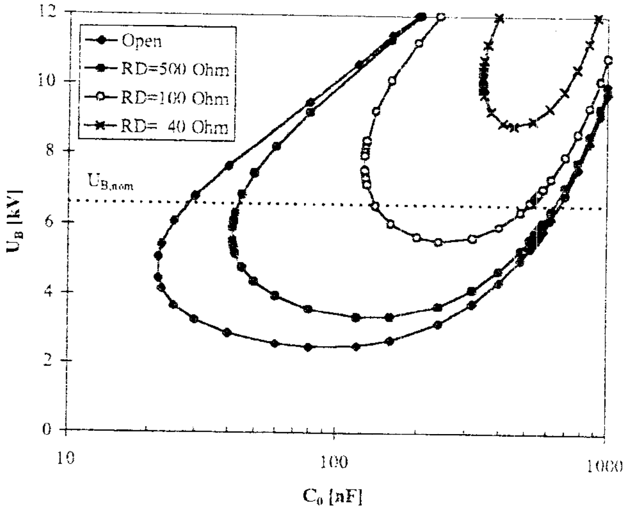
Fig.10: *Approximated inner stability domains of QP½*



5. - Damping

The three computational tools can easily be extended to take a damping resistor RD into account. This resistor is usually placed in the delta connected tertiary windings. Its effect is clearly seen on Fig.11. The stability domain of the ferroresonant oscillation (in this case QP½ which is the most critical one to damp) is shifted with lower resistor values to higher C_0- and U_B-values. The resistor to be inserted is the one that shifts the stability domain some 10% above nominal voltage over the C_0-range of interest in order to be certain that ferroresonance cannot occur in the studied configuration. The results in Fig. 11 were calculated with the harmonic balance method using the AFD-equations.

Fig.11: Damping of QP½



6. - Conclusions

In this paper, time and frequency domain methods to study ferroresonance were compared. Time domain methods suffer from the drawback that the system order increases with the number of inductive and capacitive elements in the linear part of the circuit. This restricts their use to fairly simple networks that can be expressed as a limited number of ordinary differential equations. The collocation based method is preferred over the Floquet-method since transients do not interfere with the automatic computation of the stability domains.

The harmonic balance method is very well suited to determine ferroresonant oscillations and their stability domains. The system order is reduced using Thévenin-equivalents and the AFD-equations prove to produce good approximations for quasi-periodic oscillations.

In the near future nonlinear protective devices will be studied with this method. These devices may be needed in those cases where the thermal dissipation of the VT's due to the damping resistor becomes inadmissible. Alternates for the approximated frequency domain equations will be studied as well.

7. - References

[1] N.Germay, S.Mastero, J.Vroman, "Review of Ferroresonance Phenomena in High Voltage Power Systems and Presentation of a Voltage Transformer Model for predetermining them", *CIGRE* report 33-18, 1974

[2] N.Janssens, T.Van Craenenbroeck, D.Van Dommelen, F.Van De Meulebroeke, "Direct Calculation of the Stability Domains of Three-Phase Ferroresonance in Isolated Neutral Networks with Grounded-Neutral Voltage Transformers" , *IEEE Trans. Power Delivery*, vol.11, no.3, July 1996, pp.1546-1553

[3] C.Stuckens, P.-A.Monfils, N.Janssens, T.Van Craenenbroeck, D.Van Dommelen, " Risk of Ferroresonance in Isolated Neutral Networks and Remedies", *IEE Conference Publication No.438, CIRED*, pp.1.18.1-1.18.6, 1997

[4] B.A.Mork, D.L.Stuehm, "Application of Nonlinear Dynamics and Chaos to Ferroresonance in Distribution Systems, *IEEE Trans. on PWRD* , vol.9, no.2, pp.1009-1017, April 1994

[5] R.Seydel, *Practical Bifurcation and Stability Analysis*, Springer Verlag, 1994

[6] E.J.Doedel, J.P.Kernévez, "AUTO: Software for Continuation and Bifurcation Problems in Ordinary Differential Equations", *Applied Mathematics Report*, California Institute of Technology, 1986

[7] E.J.Doedel, H.B.Keller, J.P.Kernévez, "Numerical Analysis and Control of Bifurcation Problems (II): Bifurcations in Infinite Dimensions, *Int.Journ. of Bifurcation and Chaos*, vol.1, no.4, 1996, pp.745-772

[8] T.Van Craenenbroeck, W.Michiels, D.Van Dommelen, K.Lust: "Bifurcation Analysis of Three-Phase Ferroesonant Oscillations in Ungrounded Power Systems", PE-152-PWRD-0-06-1998, to be published in *IEEE Trans. on PWRD*

[9] C.Kieny, "Ferroresonance Study using Galerkin Method with Pseudo-Arclength Continuation Method", *IEEE Trans. on PWRD*, vol.6, no.4, October 1991, pp.1841-1847

[10]T.Parker, L.Chua: *Practical numerical algorithms for chaotic systems*, Springer Verlag, 1989

[11]T.Van Craenenbroeck, *Bifurcatie-analyse van driefasige ferroresonantie in elektrische netten met geïsoleerd nulpunt*, Ph.D. dissertation K.U.Leuven, ISBN 90-5682-127-X, May 1998

[12]C.Piccardi, "Bifurcations of Limit Cycles in Periodically Forced Nonlinear Systems: The Harmonic Balance Approach", *IEEE Trans. On Circuits and Systems I*, vol.41, no.4, pp.315-320, April 1994