Advancing LPV data-driven modeling and control via LS-SVMs

Dario Piga
Control Systems Group
Eindhoven University of Technology
Eindhoven, The Netherlands

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Technische Universiteit
Eindhoven
University of Technology

Where innovation starts

Outline

1. Instrumental-Variable based extension of LS-SVM
2. Identification of LPV systems under general noise conditions
3. Model-free controller design for LPV systems

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2. Identification of LPV systems under general noise conditions
3. Model-free controller design for LPV systems

IV-SVM: a motivating example

Data-generating system

$$y(k) = a\,y(k-1) + f^{o}(y(k-2)) + g^{o}(u(k)) + b\,u(k-1)$$

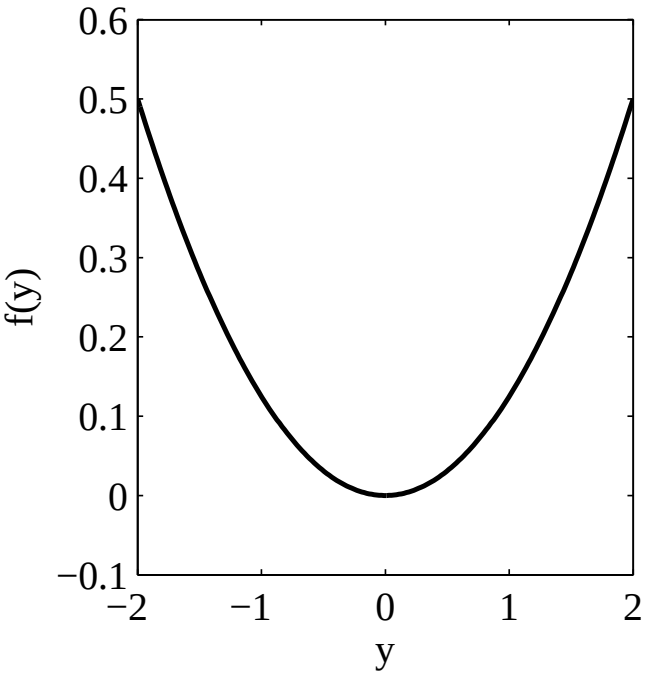
$$\text{with } a = -0.7, \quad b = -0.4$$

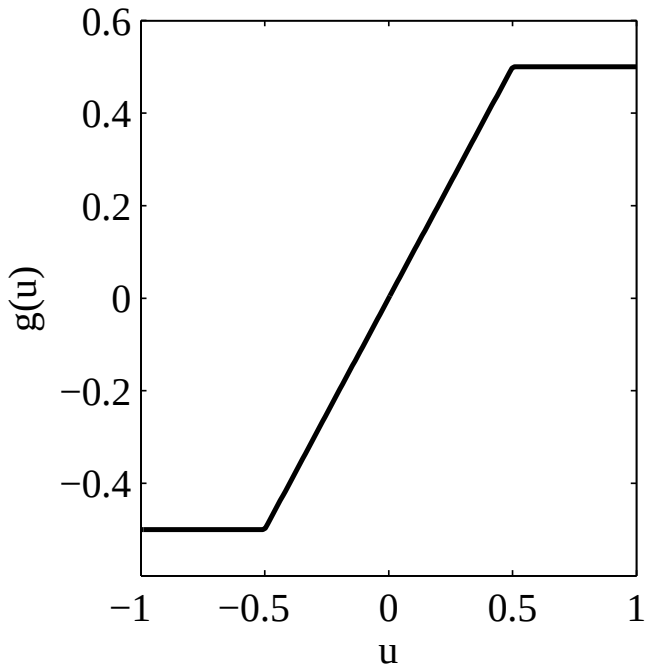
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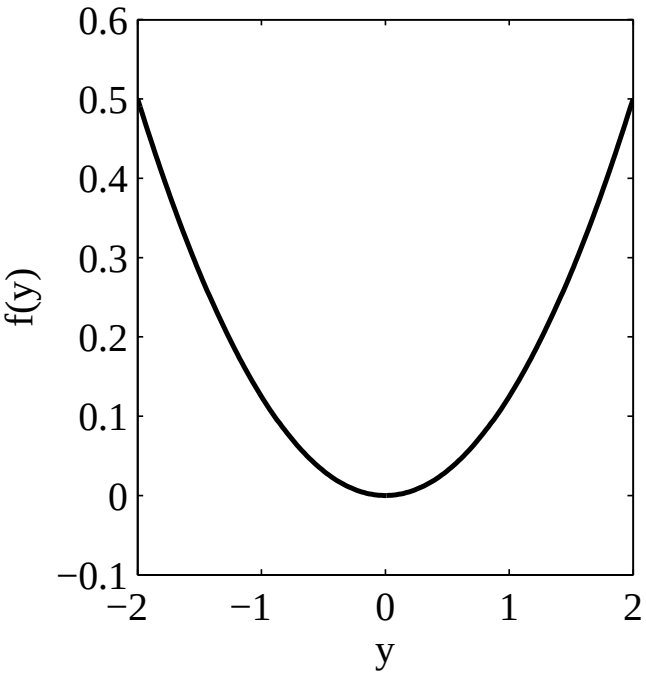


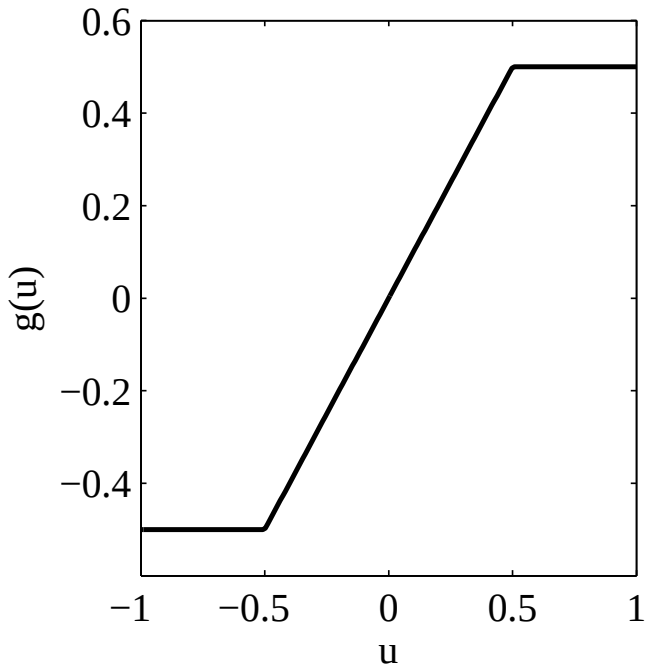
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$$y(k) = a\,y(k-1) + f^{o}(y(k-2)) + g^{o}(u(k)) + b\,u(k-1) + v_0(k)$$

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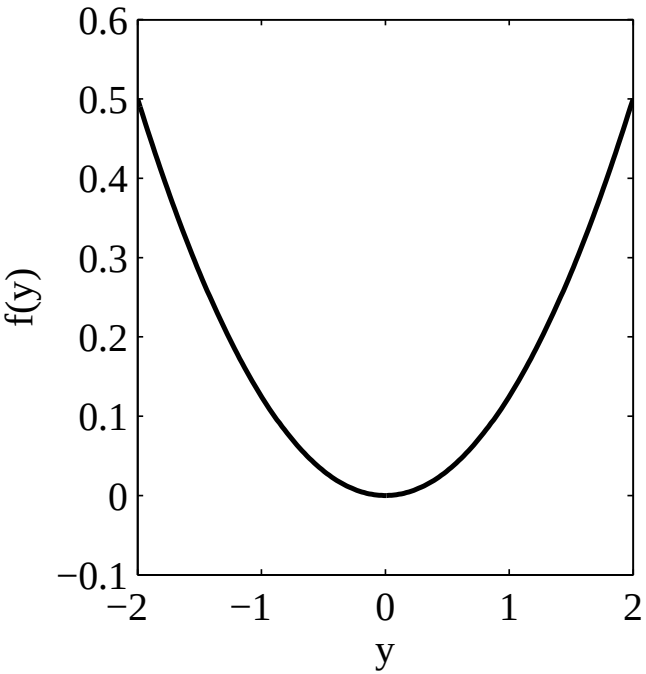
v_0 colored noise

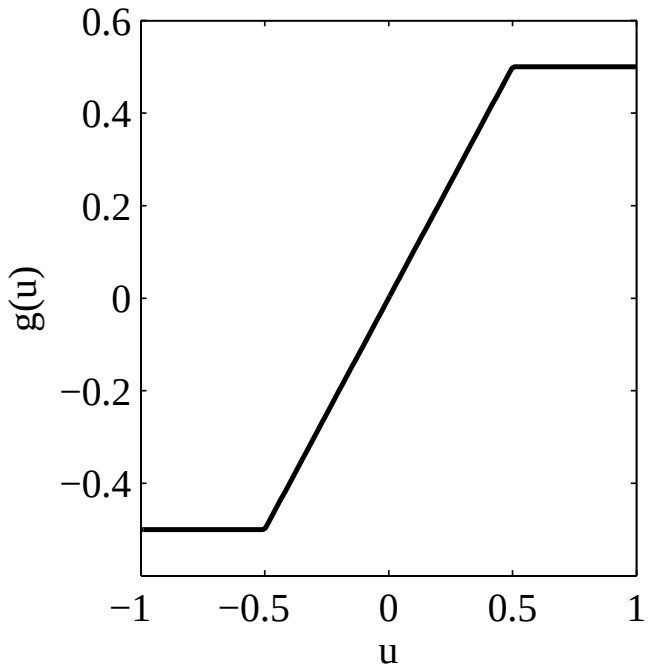
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v_0 colored noise

SNR=11 dB

N=1000

IV-SVM: a motivating example

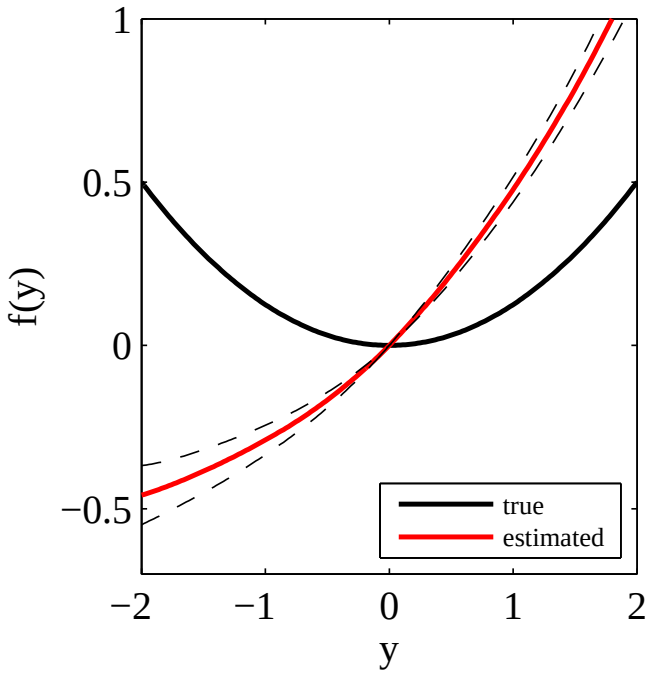
LS-SVM estimate

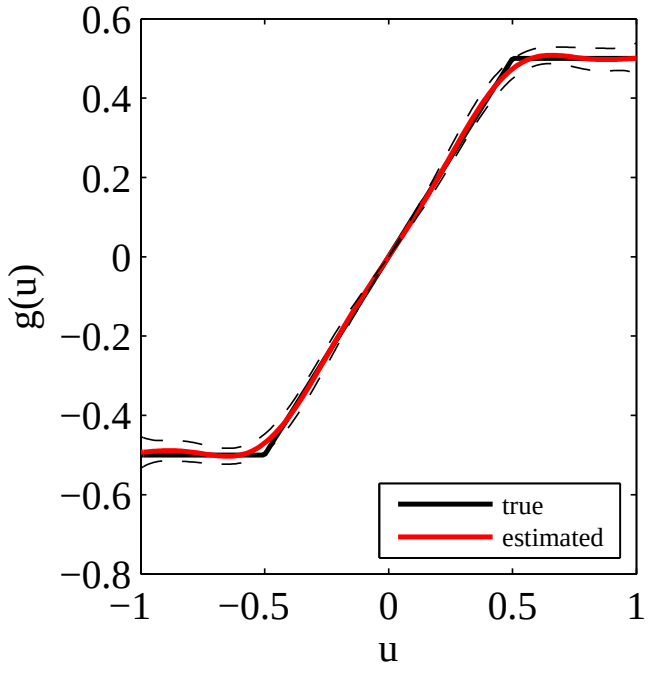
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b	-0.4	-0.6983	0.0320

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Handicap of the LS estimate

Data-generating system

$$y(k) = \phi^\top(k)\theta_o + v_o(k) \qquad \phi(k) = \left[y(k-1) \ \ldots \ y(k-n_a) \ u(k) \ \ldots \ u(k-n_b)\right]^\top$$

Handicap of the LS estimate

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$$\hat{\theta}_{LS} = \left(\Phi^\top\Phi\right)^{-1}\Phi^\top\left(\Phi\theta_o + V_o\right) = \theta_o + \left(\Phi^\top\Phi\right)^{-1}\Phi^\top V_o$$

Handicap of the LS estimate

LS estimate

$$\hat{\theta}_{\mathrm{LS}} = \theta_o + \left(\Phi^\top \Phi\right)^{-1} \Phi^\top V_{\mathrm{o}}$$

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$$\lim_{N\to\infty} \frac{1}{N}\sum_{k=1}^{N} \phi(k) v_{\mathrm{o}}(k) \neq 0 \quad \text{if} \quad v_{\mathrm{o}}(k) \text{ is colored.}$$

Instrumental Variable (IV) approach

IV estimate

$$\min_{\theta} \sum_{k=1}^{N} \left(y(k) - \phi^{\top}(k)\theta\right)^2$$

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- $\dim(z(k)) = \dim(\phi(k))$ (just for the sake of exposition)
- $z(k)$ is called the instrument

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IV estimate

Consistency property

$$\hat{\theta}_{\mathrm{IV}} = \left(Z^\top \Phi\right)^{-1} Z^\top Y, \qquad Y = \Phi\theta_{\mathrm{o}} + V_{\mathrm{o}}$$

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IV estimate

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$$\begin{aligned}
\hat{\theta}_{\mathrm{IV}} &= \left(Z^\top \Phi\right)^{-1} Z^\top Y = \\
&= \left(Z^\top \Phi\right)^{-1} Z^\top \left(\Phi\theta_{\mathrm{o}} + V_{\mathrm{o}}\right) = \\
&= \theta_o + \left(Z^\top \Phi\right)^{-1} Z^\top V_{\mathrm{o}} \\
\lim_{N\to\infty} \hat{\theta}_{\mathrm{IV}} &= \theta_o + \lim_{N\to\infty} \left(\frac{1}{N} Z^\top \Phi\right)^{-1} \lim_{N\to\infty} \frac{1}{N} Z^\top V_{\mathrm{o}} = \\
&= \theta_o + \lim_{N\to\infty} \left(\frac{1}{N} Z^\top \Phi\right)^{-1} \lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} z(k) v_{\mathrm{o}}(k)
\end{aligned}$$

$$\lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} z(k) v_{\mathrm{o}}(k) = \mathbb{E}\left[z(k) v_{\mathrm{o}}(k)\right]$$

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- In order to minimize the variance of the estimate, the instrument should be correlated with the noise-free regressor, i.e.,

$$z_{\mathrm{o}}(k) = \phi_{\mathrm{o}}(k) = \begin{bmatrix} y_{\mathrm{o}}(k-1) \\ \vdots \\ y_{\mathrm{o}}(k-n_{\mathrm{a}}) \\ u(k) \\ \vdots \\ u(k-n_{\mathrm{b}}) \end{bmatrix}$$

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- The noise-free regressor cannot be computed!

IV estimate

Recursive approximation of the instruments

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IV estimate

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4. Compute the IV estimate $\hat{\theta}_{\mathrm{IV}}$
5. Repeat from step 2

IV-SVM

Data-generating system

$$y(k) = f^{o}(x(k)) + v_{o}(k) =$$

$$\text{with } x(k) = \left[y(k-1) \ \ldots y(k-n_{a})\, u(k) \ \ldots \ u(k-n_{b})\right]$$

IV-SVM

Data-generating system

$$y(k) = f^{\mathrm{o}}(x(k)) + v_{\mathrm{o}}(k) = \phi^{\top}(x(k))\theta_{\mathrm{o}} + v_{\mathrm{o}}(k)$$

with $x(k) = \left[y(k-1) \ \ldots y(k-n_{\mathrm{a}})\, u(k) \ \ldots \ u(k-n_{\mathrm{b}})\right]$

$\phi : \mathbb{R}^{n_x} \to \mathbb{R}^{n_\theta}$ (mapping to a feature space))

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LS-SVM: primal problem

$$\hat{\theta} = \min_{\theta, e} \|\theta\|^2 + \gamma \sum_{k=1}^{N} e^2(k)$$

$$\text{s.t. } e(k) = y(k) - \phi^{\top}(x(k))\theta$$

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$$\hat{\theta} = \min_{\theta, e} \|\theta\|^2 + \gamma \| \sum_{k=1}^{N} z(k)e(k) \|^2$$

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- $z(k)$ chosen by the user to satisfy the following condition $\mathbb{E}[z(k)v_o(k)] = 0$
- $\dim(z(k)) = \dim(\phi(x(k)))$

IV-SVM

$$\phi(x(k))$$

$$x(k) = \begin{bmatrix} y(k-1) \\ \vdots \\ y(k-n_a) \\ u(k) \\ \vdots \\ u(k-n_b) \end{bmatrix}$$

IV-SVM

$$\phi(x(k)) \quad \rightarrow \quad z(k) = \phi(\xi(k))$$

$$x(k) = \begin{bmatrix} y(k-1) \\ \vdots \\ y(k-n_a) \\ u(k) \\ \vdots \\ u(k-n_b) \end{bmatrix} \quad \rightarrow \quad \xi(k) = \begin{bmatrix} y_o(k-1) \\ \vdots \\ y_o(k-n_a) \\ u(k) \\ \vdots \\ u(k-n_b) \end{bmatrix}$$

IV-SVM

IV-SVM: primal problem

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IV-SVM: dual problem

$$\mathcal{L}(\theta, e, \alpha) = \|\theta\|^2 + \gamma \left\| \sum_{k=1}^{N} z(k)e(k) \right\|^2 + \sum_{k=1}^{N} \alpha_k \left(e(k) - y(k) - \phi^\top(x(k)) \right)$$

$$\frac{\partial \mathcal{L}}{\partial \theta} = 0, \quad \frac{\partial \mathcal{L}}{\partial e} = 0 \quad \frac{\partial \mathcal{L}}{\partial \alpha} = 0$$

IV-SVM

$$\alpha = \left(\Phi\Phi^{\top} + \gamma^{-1} I_N \right)^{-1} Y$$

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$$\Phi\Phi^{\top} = \begin{bmatrix} \phi^{\top}(x(1)) \\ \vdots \\ \phi^{\top}(x(N)) \end{bmatrix} \begin{bmatrix} \phi(x(1)) & \cdots & \phi(x(N)) \end{bmatrix}$$

IV-SVM

$$\alpha = \left(\Phi\Phi^\top + \gamma^{-1} I_N \right)^{-1} Y$$

$$\Phi\Phi^\top = \begin{bmatrix} \phi^\top(x(1)) \\ \vdots \\ \phi^\top(x(N)) \end{bmatrix} \begin{bmatrix} \phi(x(1)) & \cdots & \phi(x(N)) \end{bmatrix} \rightarrow \left[\Phi\Phi^\top\right]_{i,j} = \mathcal{K}(x(i), x(j))$$

$$x(k) = \begin{bmatrix} y(k-1) & \ldots y(k-n_\mathrm{a}) & u(k) & \ldots & u(k-n_\mathrm{b}) \end{bmatrix}$$

IV-SVM

$$\alpha = \left(Z Z^\top \Phi \Phi^\top + \gamma^{-1} I_N \right)^{-1} Z Z^\top Y$$

$$\Phi \Phi^\top = \begin{bmatrix} \phi^\top(x(1)) \\ \vdots \\ \phi^\top(x(N)) \end{bmatrix} \begin{bmatrix} \phi(x(1)) & \cdots & \phi(x(N)) \end{bmatrix} \rightarrow \left[\Phi \Phi^\top \right]_{i,j} = \mathcal{K}(x(i), x(j))$$

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$$Z Z^\top = \begin{bmatrix} \phi^\top(\xi(1)) \\ \vdots \\ \phi^\top(\xi(N)) \end{bmatrix} \begin{bmatrix} \phi(\xi(1)) & \cdots & \phi(\xi(N)) \end{bmatrix}$$

$$x(k) = \begin{bmatrix} y(k-1) & \ldots y(k-n_\mathrm{a}) \, u(k) & \ldots & u(k-n_\mathrm{b}) \end{bmatrix}$$

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$$\xi(k) = \begin{bmatrix} y_\mathrm{o}(k-1) & \ldots y_\mathrm{o}(k-n_\mathrm{a}) \, u(k) & \ldots & u(k-n_\mathrm{b}) \end{bmatrix}$$

IV-SVM

$$\alpha = \left(Z Z^\top \Phi \Phi^\top + \gamma^{-1} I_N \right)^{-1} Z Z^\top Y$$

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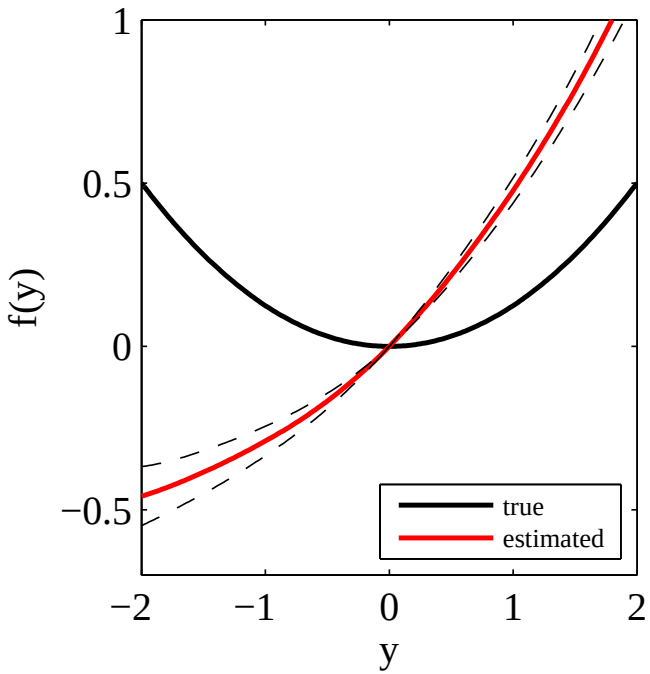
$$Z Z^\top = \begin{bmatrix} \phi^\top(\xi(1)) \\ \vdots \\ \phi^\top(\xi(N)) \end{bmatrix} \begin{bmatrix} \phi(\xi(1)) & \cdots & \phi(\xi(N)) \end{bmatrix} \rightarrow \left[Z Z^\top \right]_{i,j} = \mathcal{K}(\xi(i), \xi(j))$$

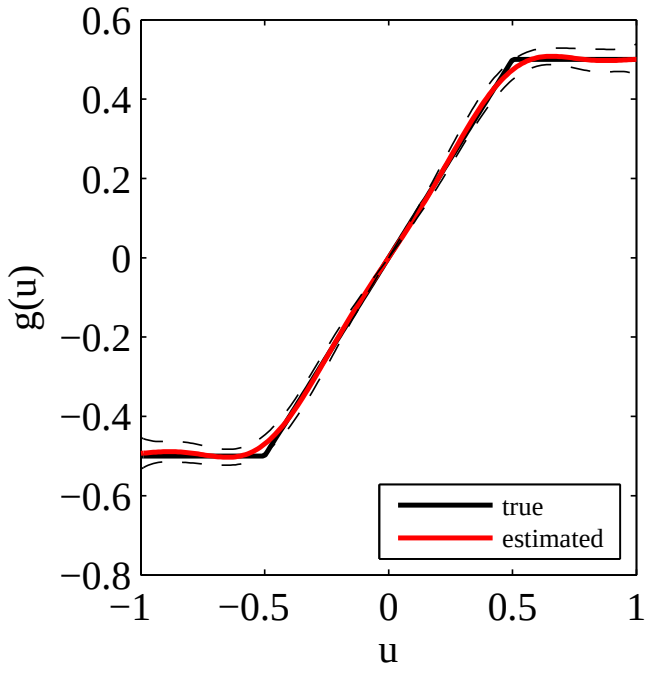
$$\hat{f}(\tilde{x}) = \sum_{k=1}^{N} \alpha_k \mathcal{K}(x(k), \tilde{x})$$

IV-SVM: a motivating example

LS-SVM estimate

	True value	mean	std
a	-0.7	-0.2684	0.0451
b	-0.4	-0.6983	0.0320

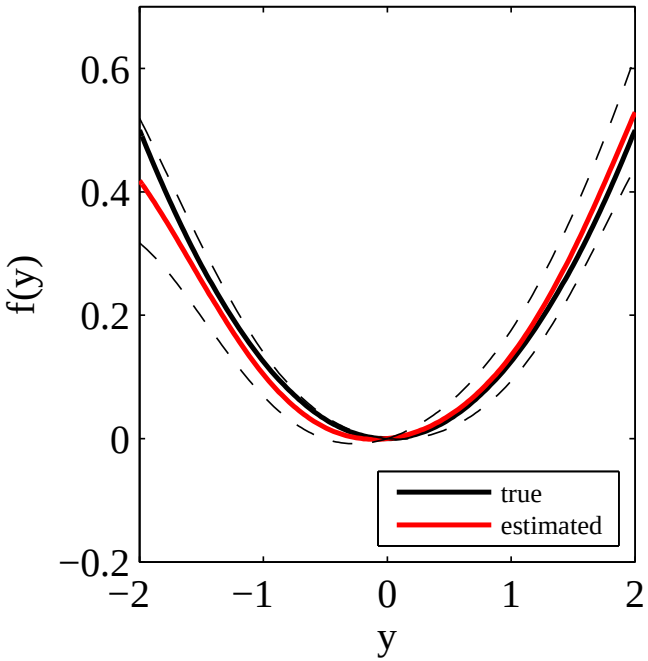


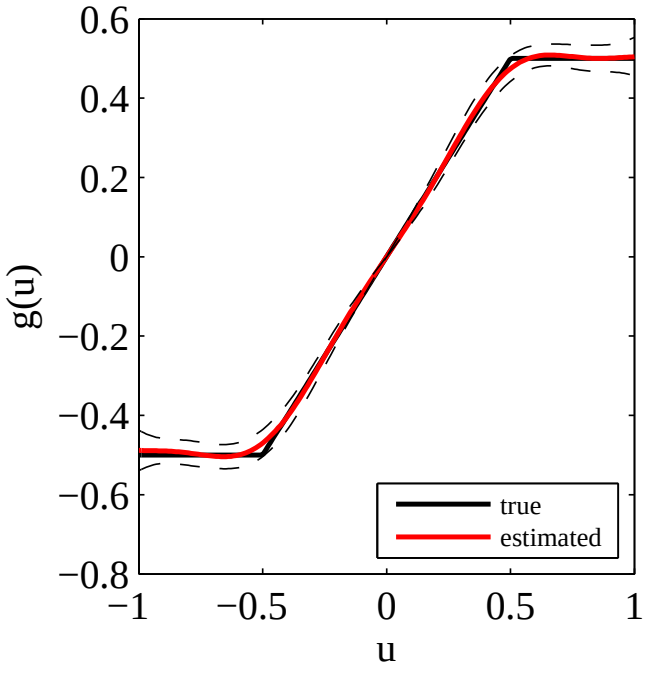


IV-SVM: a motivating example

IV-SVM estimate

	True value	mean	std
a	-0.7	-0.6920	0.0459
b	-0.4	-0.4062	0.0371





Outline

1. Instrumental-Variable based extension of LS-SVM
2. **Identification of LPV systems under general noise conditions**
3. Model-free controller design for LPV systems

LPV-OE models

Data-generating system

$$y_o(k) = \sum_{i=1}^{n_a} a_i^o(p(k))y_o(k-i) + \sum_{j=0}^{n_b} b_j^o(p(k))u_o(k-j)$$

LPV-OE models

Data-generating system

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$$y(k) = y_o(k) + e_o(k), \quad e_o(k) \text{ white}$$

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LPV-OE models

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$v_o(k)$ is zero mean, but it is not white!

LPV identification

Data-generating system

$$y(k) = \sum_{i=1}^{n_{\mathrm{H}}} h_i^{\mathrm{o}}(p(k)) x_i(k) + v_o(k)$$

$$x(k) = \left[y(k-1) \ \ldots \ y(k-n_{\mathrm{a}}) \, u(k) \ \ldots u(k-n_{\mathrm{b}}) \right]$$

LPV identification

Data-generating system

$$y(k) = \sum_{i=1}^{n_{\mathrm{H}}} h_i^{\mathrm{o}}(p(k)) x_i(k) + v_o(k) = \sum_{i=1}^{n_{\mathrm{H}}} \phi_i^{\top}(p(k)) \theta_i x_i(k) + v_o(k)$$

$$x(k) = \left[y(k-1) \ \ldots \ y(k-n_{\mathrm{a}}) \, u(k) \ \ldots u(k-n_{\mathrm{b}}) \right]$$

LPV identification

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LPV LS-SVM

$$\min_{\theta, v} \sum_{i=1}^{n_{\mathrm{H}}} \| \theta_i \|^2 + \gamma \sum_{k=1}^{N} v^2(k)$$

$$\text{s.t. } v(k) = y(k) - \sum_{i=1}^{n_{\mathrm{H}}} \phi^{\top}(p(k)) x_i(k) \theta_i$$

LPV identification

LPV IV-SVM

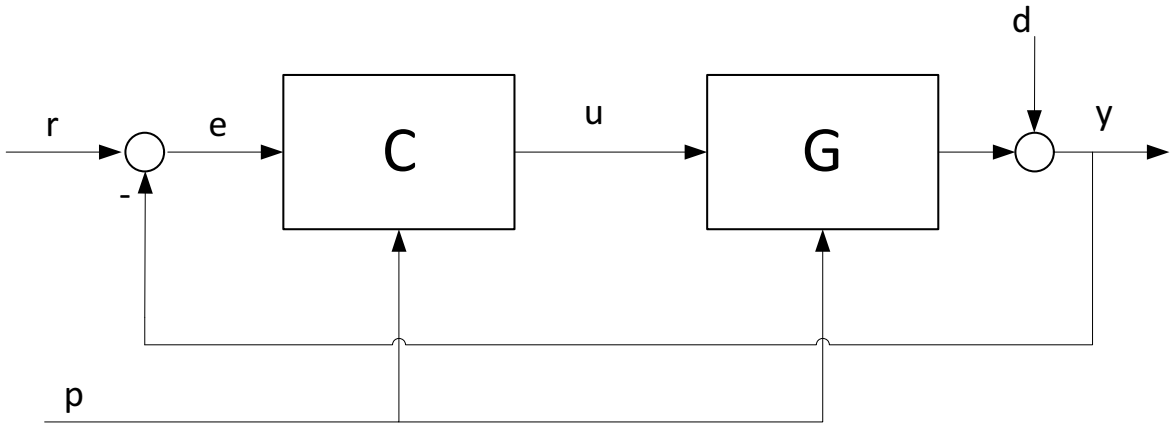
$$\min_{\theta, v} \sum_{i=1}^{n_{\mathrm{H}}} \|\theta_i\|^2 + \gamma \left\| \sum_{k=1}^{N} \xi(k) v(k) \right\|^2$$

$$\text{s.t. } v(k) = y(k) - \sum_{i=1}^{n_{\mathrm{H}}} \phi_i^{\top}(p(k)) x_i(k) \theta_i$$

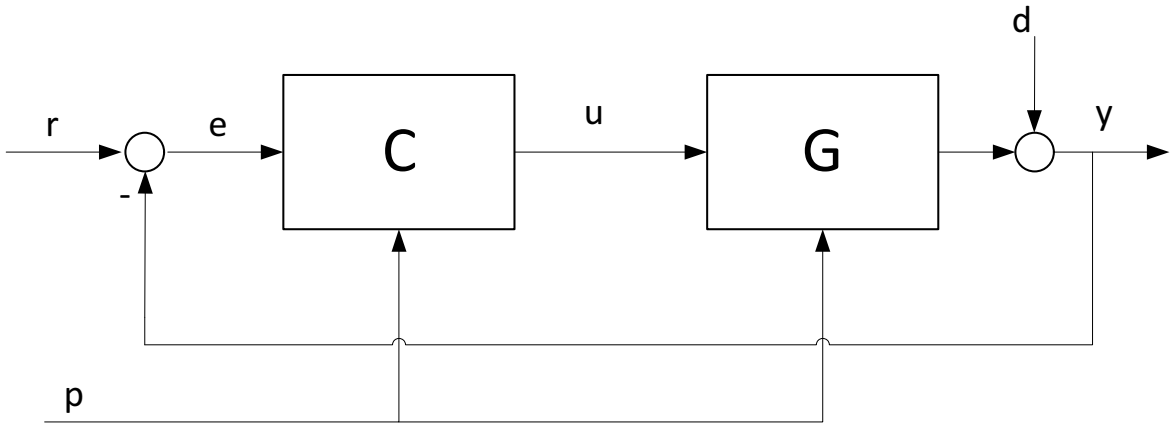
Outline

1. Instrumental-Variable based extension of LS-SVM
2. Identification of LPV systems under general noise conditions
3. **Model-free controller design for LPV systems**

Problem description

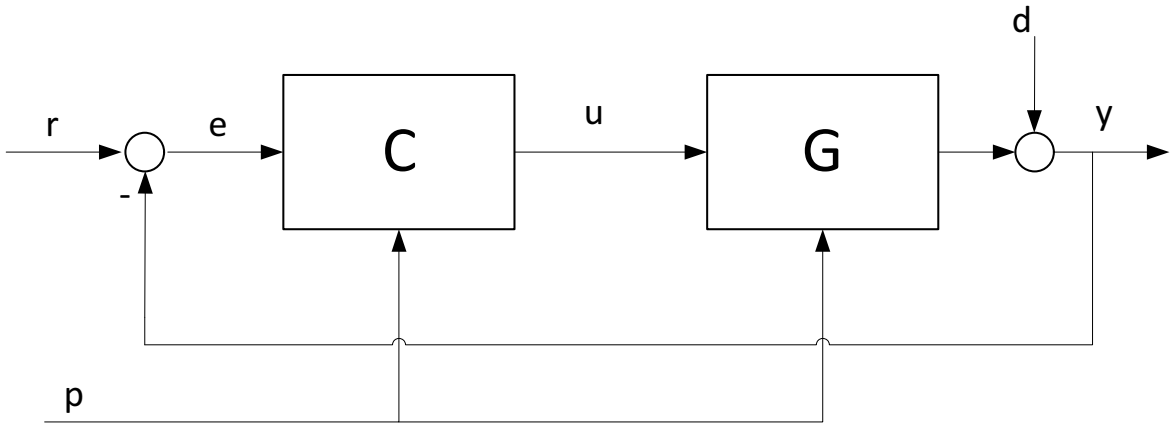


Problem description



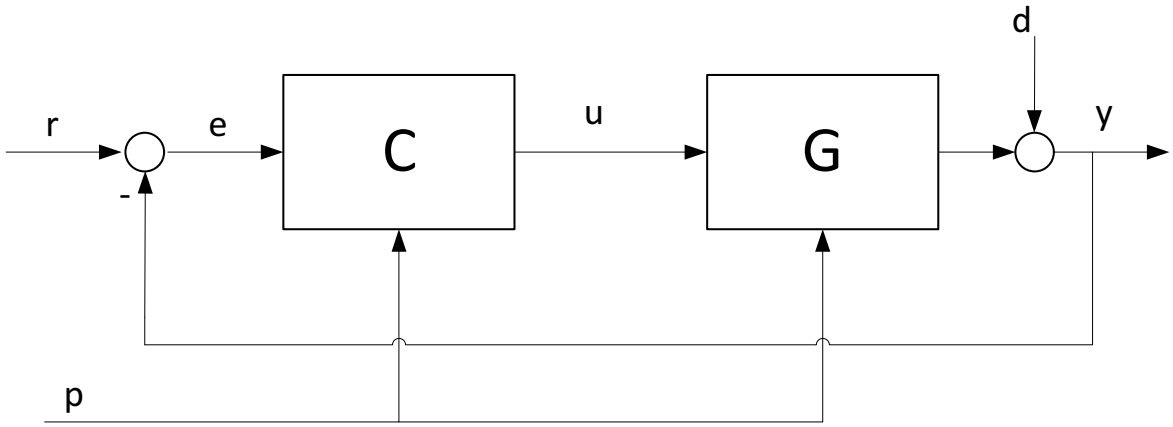
- Design an LPV controller in order to achieve a desired closed-loop behaviour

Problem description



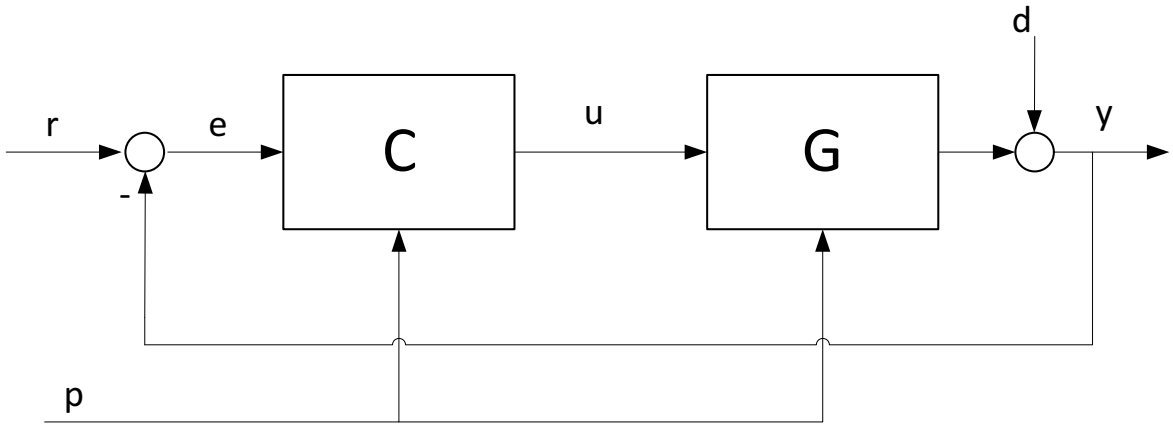
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- The model of the plant is not available

Problem description



- Design an LPV controller in order to achieve a desired closed-loop behaviour
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- A set of observations $\{u(k),\, y(k),\, p(k)\}_{k=1}^{N}$ is available

Problem description



- Design an LPV controller in order to achieve a desired closed-loop behaviour
- The model of the plant is not available
- A set of observations $\{u(k),\, y(k),\, p(k)\}_{k=1}^{N}$ is available
- The controller should be designed directly from the data (without deriving a model of the plant)

Advantages

- Avoid the cost- and time-consuming intermediate phase of plant modeling

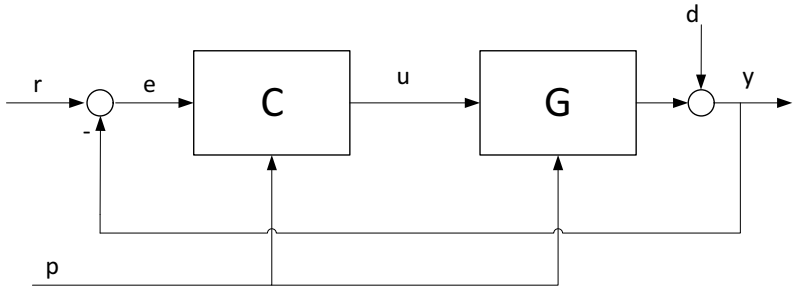
Advantages

- Avoid the cost- and time-consuming intermediate phase of plant modeling
- Fully automated procedure for control design

Advantages

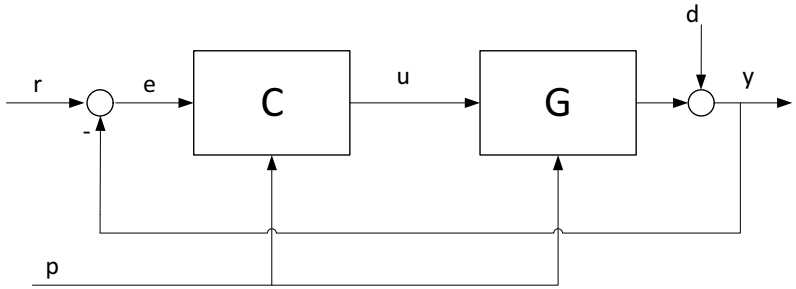
- Avoid the cost- and time-consuming intermediate phase of plant modeling
- Fully automated procedure for control design
- The degrees of freedom in the selection of the controller are directly used to achieve the final control goal. The plant dynamics, which are relevant to achieve the specified control objectives, are directly chosen by the data.

Virtual reference feedback tuning



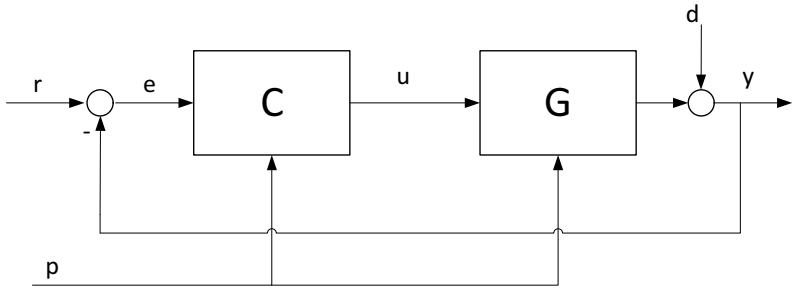
- Collect a sequence of data $\{u(k), y(k), p(k)\}_{k=1}^{N}$

Virtual reference feedback tuning



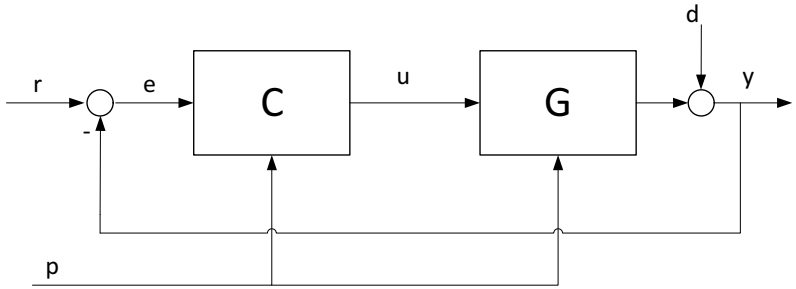
- Collect a sequence of data $\{u(k), y(k), p(k)\}_{k=1}^{N}$
- Specify a desired closed-loop behaviour M. Compute the reference signal $r_{\mathrm{v}}(k)$ such that the $y(k)$ is the output of the M for a reference signal $r_{\mathrm{v}}(k)$.

Virtual reference feedback tuning



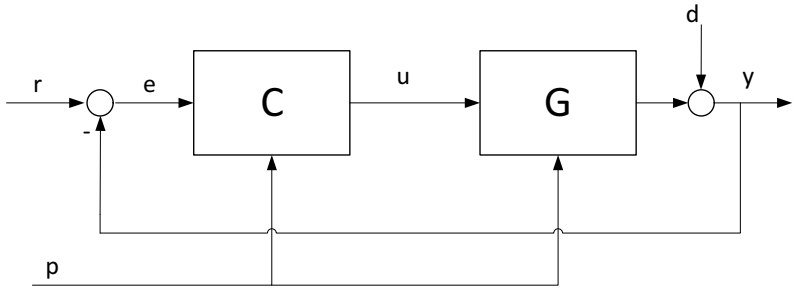
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- Compute the virtual tracking error $e_{\mathrm{v}}(k) = r_{\mathrm{v}}(k) - y(k)$. When the observed input sequence $u(k)$ is applied to the plant, the output signal will be (in a noise-free scenario) the observed sequence $y(k)$. Then, a "good" controller is the one that generates the observed sequence $u(k)$ when fed by the virtual tracking error $e_{\mathrm{v}}(k)$.

Virtual reference feedback tuning



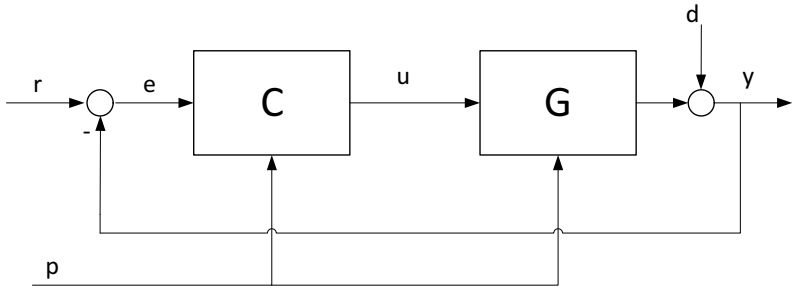
- Collect a sequence of data $\{u(k), y(k), p(k)\}_{k=1}^{N}$
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- Compute the dynamical system (i.e., the designed controller) describing the dynamic relation between $e_{\mathrm{v}}(k)$ and $u(k)$.

Virtual reference feedback tuning



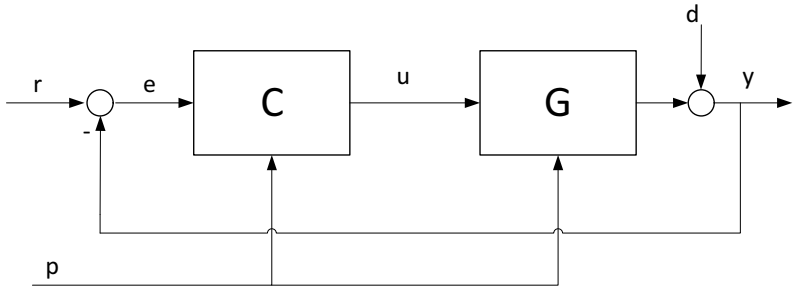
$$u(k) = \sum_{i=1}^{n_a} a_i(p)u(k-1) + \sum_{j=0}^{n_b} b_j(p)e_v(k-j)$$

Virtual reference feedback tuning



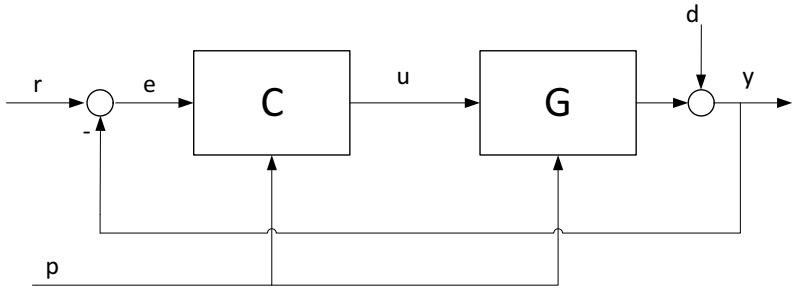
$$u(k) = \sum_{i=1}^{n_a} a_i(p)u(k-1) + \sum_{j=0}^{n_b} b_j(p)\,(r_v(k-j) - y_o(k-j))$$

Virtual reference feedback tuning



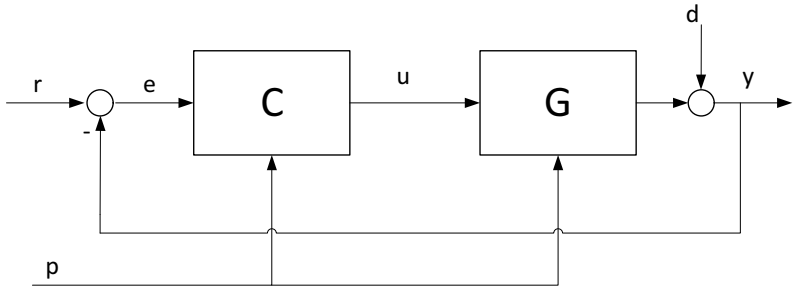
$$u(k) = \sum_{i=1}^{n_a} a_i(p)u(k-1) + \sum_{j=0}^{n_b} b_j(p)\,(r_v(k-j) - y_o(k-j)) \quad r_v(k) = M^{\dagger}(q^{-1})y_o(k)$$

Virtual reference feedback tuning



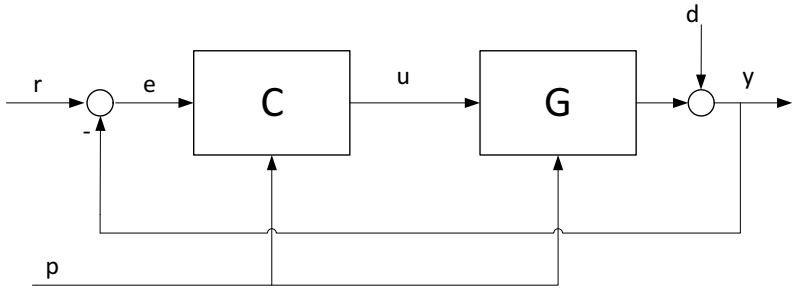
$$u(k) = \sum_{i=1}^{n_a} a_i(p)u(k-1) + \sum_{j=0}^{n_b} b_j(p)\left(M^{\dagger}(q^{-1}) - 1\right) y_o(k)$$

Virtual reference feedback tuning



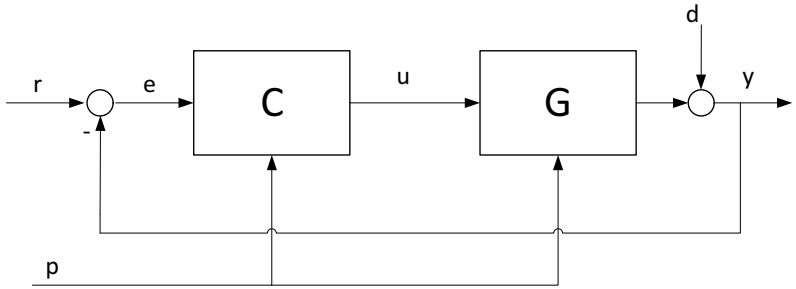
$$u(k) = \sum_{i=1}^{n_a} a_i(p)u(k-1) + \sum_{j=0}^{n_b} b_j(p)\left(M^{\dagger}(q^{-1}) - 1\right)(y(k) - d(k))$$

Virtual reference feedback tuning



$$u(k) = \sum_{i=1}^{n_a} a_i(p)\,u(k-1) + \sum_{j=0}^{n_b} b_j(p)\left(M^{\dagger}(q^{-1}) - 1\right) y(k) - \underbrace{\sum_{j=0}^{n_b} b_j(p)\left(M^{\dagger}(q^{-1}) - 1\right) d(k)}_{v(k)}$$

Virtual reference feedback tuning

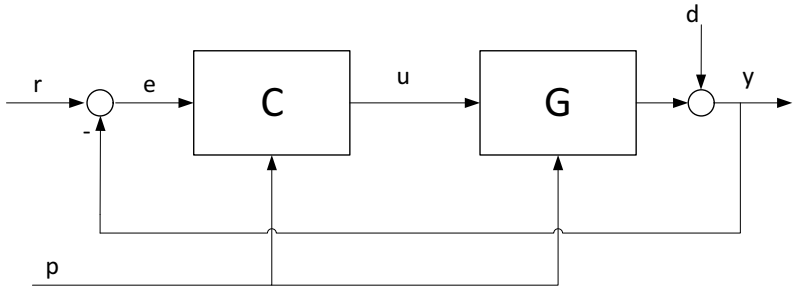


$$u(k) = \sum_{i=1}^{n_a} a_i(p)u(k-1) + \sum_{j=0}^{n_b} b_j(p)\left(M^{\dagger}(q^{-1}) - 1\right) y(k) - \underbrace{\sum_{j=0}^{n_b} b_j(p)\left(M^{\dagger}(q^{-1}) - 1\right) d(k)}_{v(k)}$$

$$\min_{a(p),b(p),v} \sum_{k=1}^{N} v^2(k)$$

$$\text{s.t.} v(k) = \sum_{i=1}^{n_a} a_i(p)u(k-1) + \sum_{j=0}^{n_b} b_j(p)\left(M^{\dagger}(q^{-1}) - 1\right) y(k) - u(k)$$

Virtual reference feedback tuning

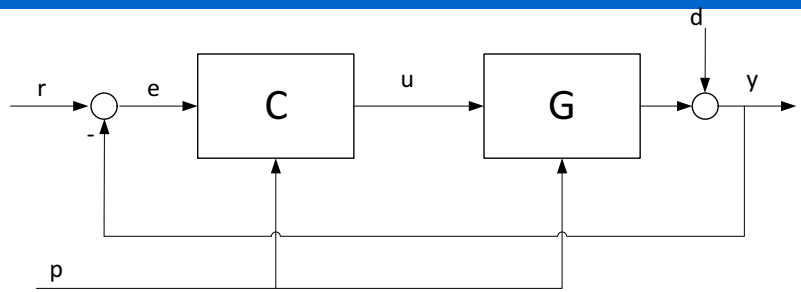


$$u(k)=\sum_{i=1}^{n_a} a_i(p)u(k-1)+\sum_{j=0}^{n_b} b_j(p)\left(M^{\dagger}(q^{-1})-1\right)y(k)-\underbrace{\sum_{j=0}^{n_b} b_j(p)\left(M^{\dagger}(q^{-1})-1\right)d(k)}_{v(k)}$$

$$\min_{a(p),b(p),v}\left\|\sum_{k=1}^{N} z(k)v(k)\right\|^2$$

$$\text{s.t.}\, v(k)=\sum_{i=1}^{n_a} a_i(p)u(k-1)+\sum_{j=0}^{n_b} b_j(p)\left(M^{\dagger}(q^{-1})-1\right)y(k)-u(k)$$

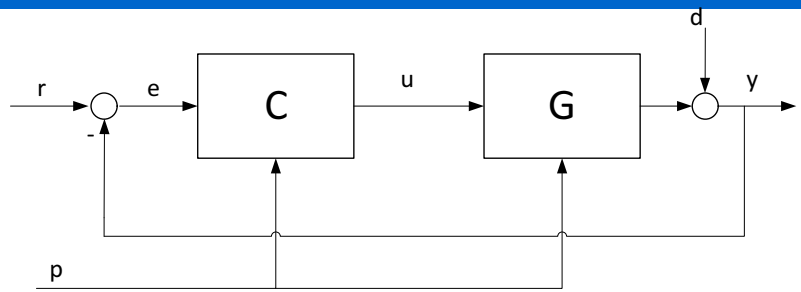
Example



(unknown) Plant

$$y(k) = p(k)y(k-1) + u(k)$$

Example

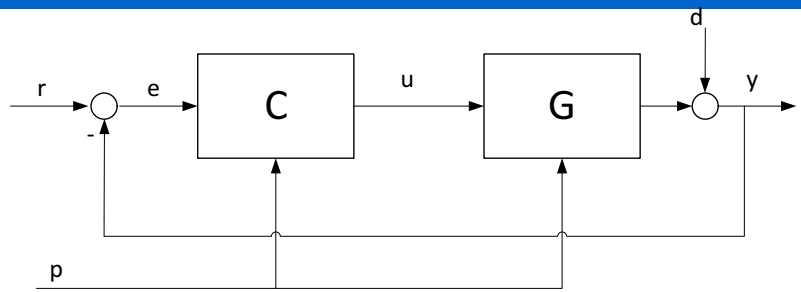


(unknown) Plant

$$y(k) = p(k)y(k-1) + u(k)$$

- Generate a data sequence $\{u(k),\ p(k),\ y(k)\}_{k=1}^{N}$, $N = 1000$.

Example

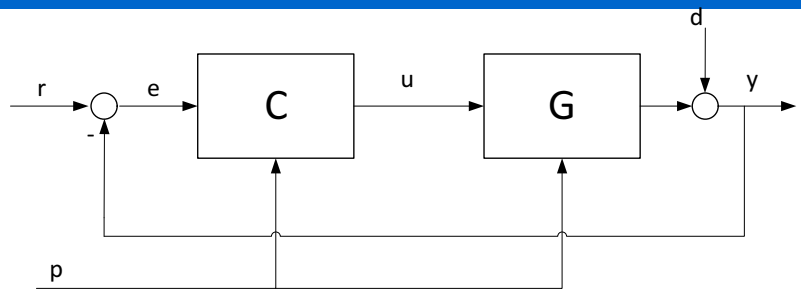


(unknown) Plant

$$y(k) = p(k)y(k-1) + u(k)$$

- Generate a data sequence $\{u(k),\ p(k),\ y(k)\}_{k=1}^{N}$, $N = 1000$.
- $u(k) \sim \mathcal{U}(-1, 1)$, $p(k) = 0.4 \sin(0.04\pi k)$,

Example

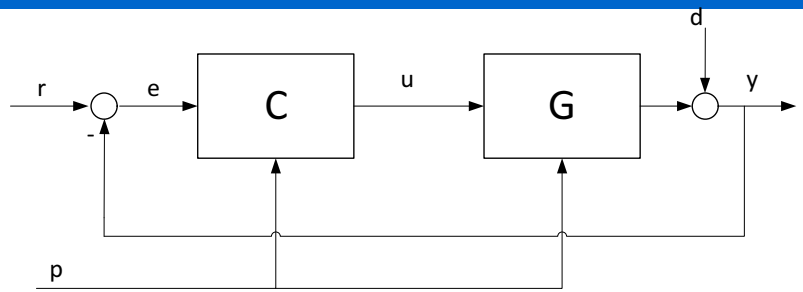


(unknown) Plant

$$y(k) = p(k)y(k-1) + u(k)$$

- Generate a data sequence $\{u(k),\ p(k),\ y(k)\}_{k=1}^{N}$, $N = 1000$.
- $u(k) \sim \mathcal{U}(-1, 1)$, $p(k) = 0.4 \sin(0.04\pi k)$, SNR $= 8$ dB.

Example

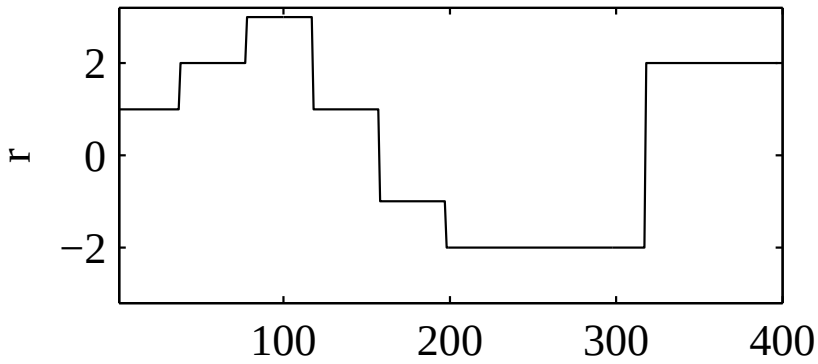


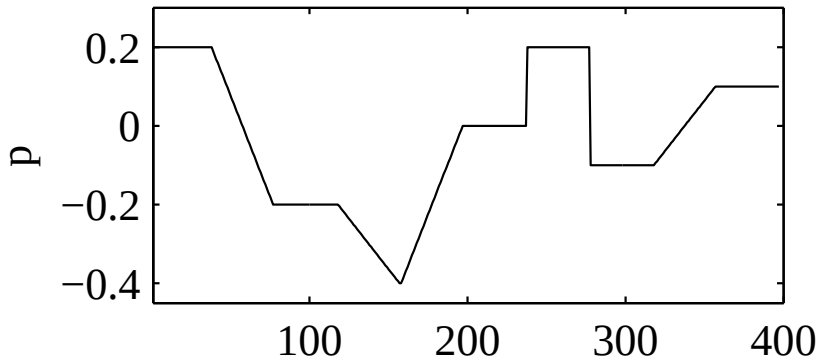
Desired closed-loop system M

$$x_{\mathrm{M}}(k+1) = \begin{bmatrix} -1 & 1 \\ -1+p(k-1)-p(k) & 1 \end{bmatrix} x_{\mathrm{M}}(k) + \begin{bmatrix} 1+p(k) \\ 1+p(k)-p(k-1) \end{bmatrix} u(k)$$

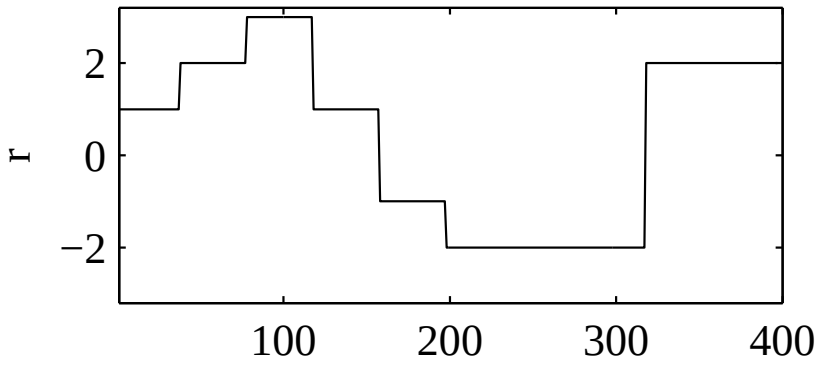
$$y_{\mathrm{M}}(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x_{\mathrm{M}}(k)$$

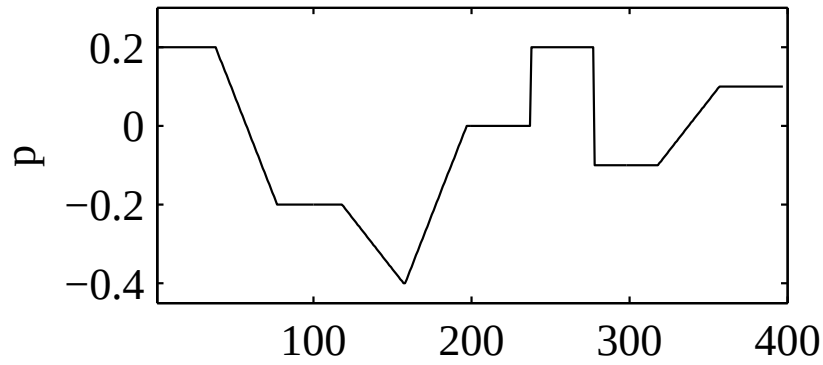
Example

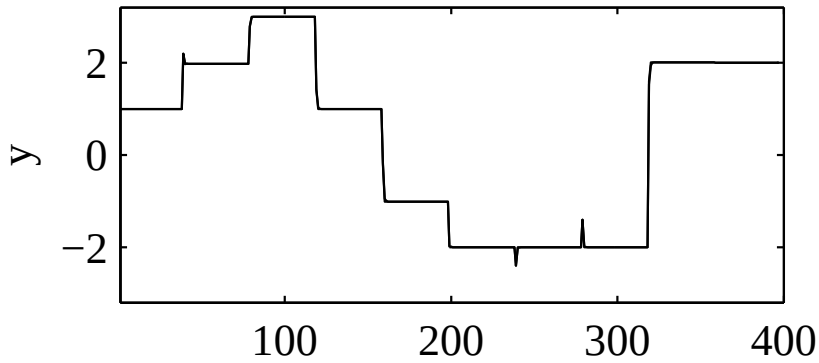


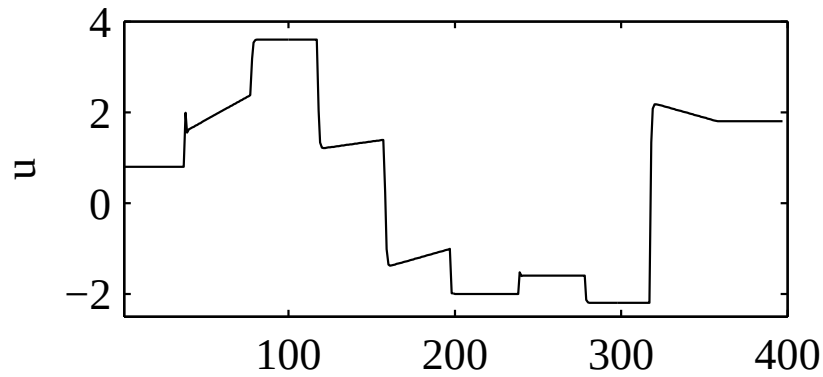


Example


r
2
0
−2
100
200
300
400


p
0.2
0
−0.2
−0.4
100
200
300
400


y
2
0
−2
100
200
300
400

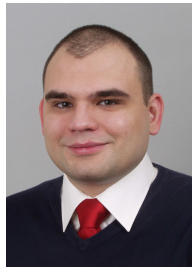

u
4
2
0
−2
100
200
300
400

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