Irredundant Lattice Piecewise Affine Representations and Their Applications in Explicit Model Predictive Control

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In this presentation, we describe the continuous piecewise affine (PWA) function with the irredundant lattice PWA representations. The domain of the continuous PWA function f is divided into nonoverlapping base regions $\mathbb{D}_1, \ldots, \mathbb{D}_N$, and in the base region $\mathbb{D}_i$, assume f equals the affine function $\ell_{\mathrm{act}(i)}$. The base region $\mathbb{D}_i,\ 1 \leq i \leq N$ is a subregion of the domain such that no other affine functions intersect with $\ell_{\mathrm{act}(i)}$ in the interior of $\mathbb{D}_i$.

The full lattice PWA representation can be expressed as $f = \max\limits_{i\in\{1,\ldots,N\}} \{ \min\limits_{j\in I_{\geq,i}} \{\ell_j\}\}$, in which $I_{\geq,i}$ is the set of indices of affine functions greater than or equal to $\ell_{\mathrm{act}(i)}$ in the base region $\mathbb{D}_i$. In the representation, $\min\limits_{j\in I_{\geq,i}} \{\ell_j\}$ and ℓ_j are called term and literal respectively. A term $\min\limits_{j\in I_i}\{\ell_j\}$ is called an implicant of f if $\min\limits_{j\in I_i}\{\ell_j\} \leq f$ in the whole domain, and in some base regions $\min\limits_{j\in I_i}\{\ell_j\} \equiv f$. From this definition, the term $\min\limits_{j\in I_{\geq,i}} \{\ell_j\}$ in the full lattice PWA representation is an implicant of f.

Based on the full lattice PWA representation, we derive the irredundant lattice PWA representation, i.e., no term or literal can be deleted from the expression without affecting the function value. Necessary and sufficient conditions for irredundancy are proposed. Besides, we propose necessary and sufficient conditions for replacing the original implicants with simpler implicants. An algorithm for obtaining an irredundant lattice PWA expression is also given.

In a worked example, the irredundant lattice PWA representation is used to express the explicit model predictive controller of a linear system, and the result turns out to be much more compact than that given by a state-of-the-art algorithm.

Keywords: continuous piecewise affine function, lattice piecewise affine representation, explicit model predictive control.