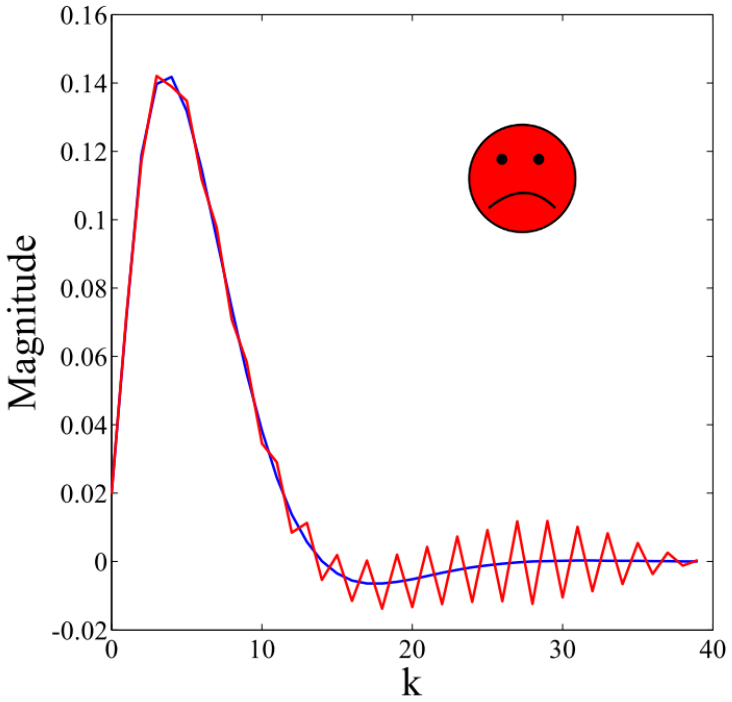
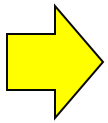
Combining regularization and SVD truncation for impulse response modeling

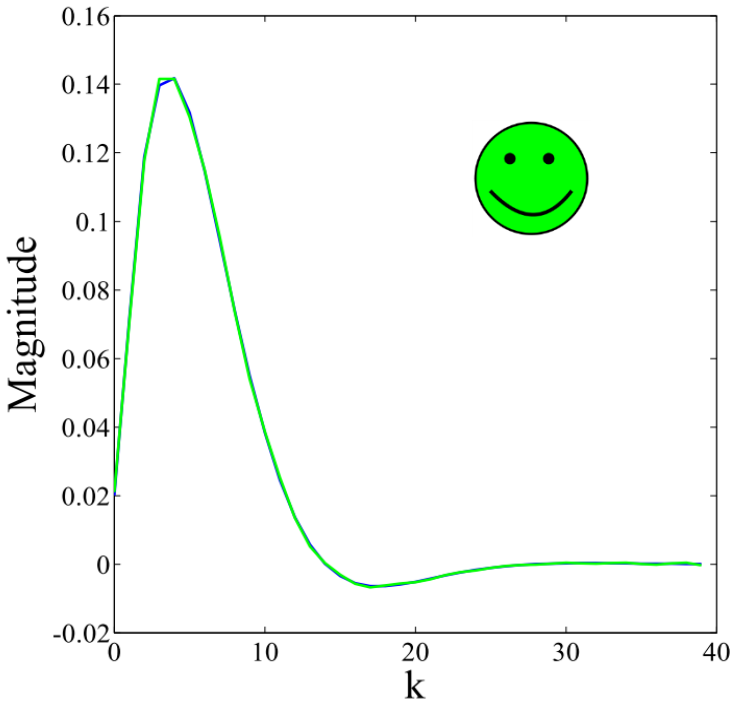
Anna Marconato*, Lennart Ljung**, Yves Rolain*, Johan Schoukens*

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** Division of Automatic Control, Dept. of Electrical Engineering, Linköping University, Sweden

Vrije Universiteit Brussel


0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
Magnitude
0
10
20
30
40
k




0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
Magnitude
0
10
20
30
40
k

Impulse response modeling

1. **Low-rank solution**
   - SVD truncation



LINK



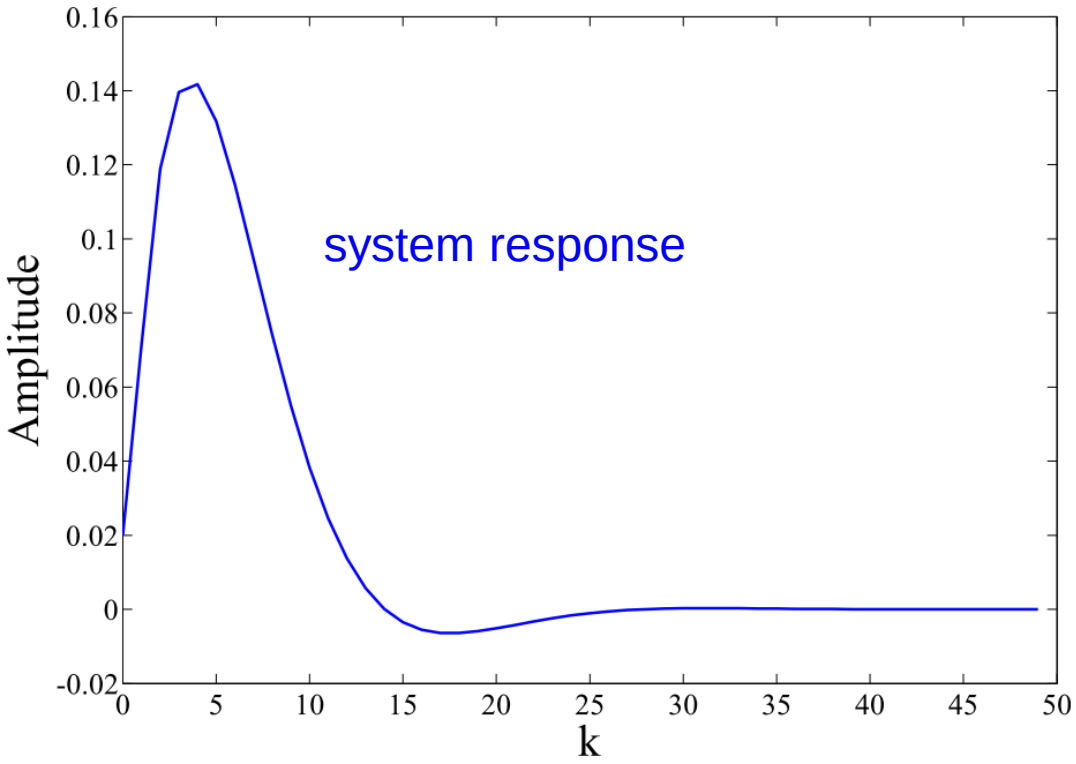
2. **Regularized solution**
   - Empirical Bayes

Impulse response modeling

1. **Low-rank solution**
   - SVD truncation

FIR modeling

$$\hat{y}(t) = \sum_{k=0}^{n_b-1} \hat{b}_k u(t-k)$$



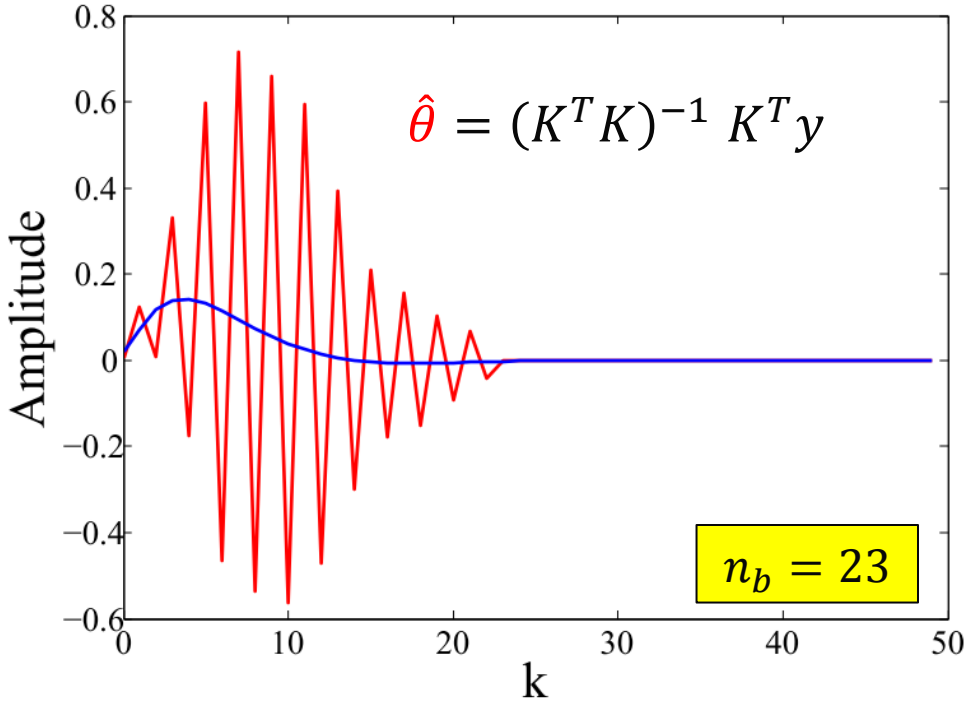
Least Squares estimate

$$\hat{y}(t) = \sum_{k=0}^{n_b-1} \hat{b}_k u(t-k)$$



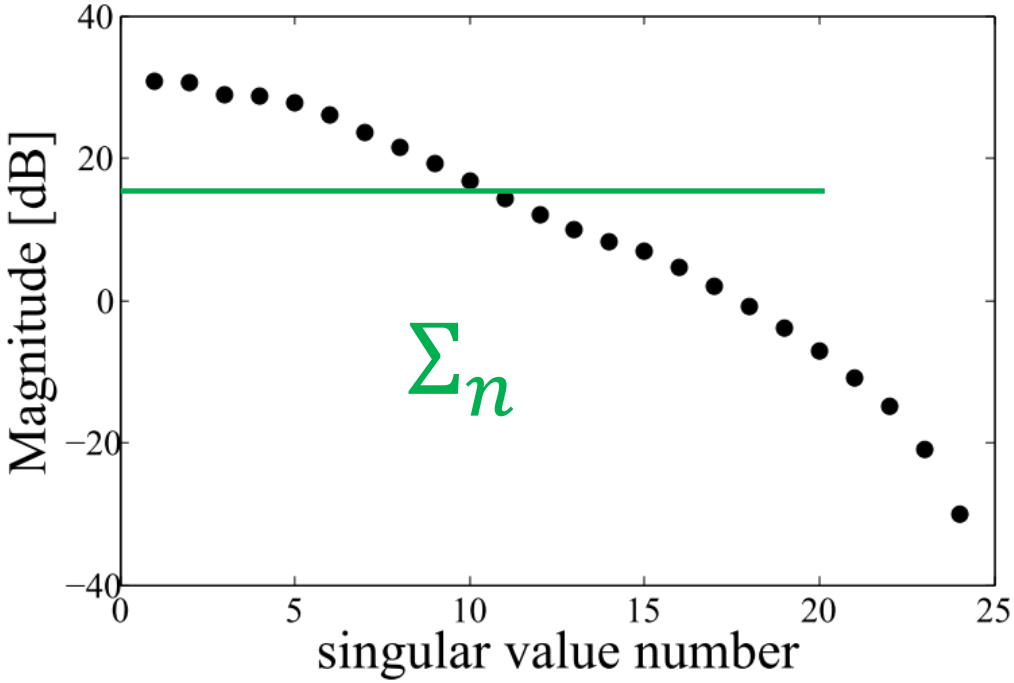
$$\hat{b} = \hat{\theta} = \arg\min_{\theta} \|y - K\theta\|^2 = (K^T K)^{-1} K^T y$$

Colored noise excitation



A possible solution: SVD truncation

$$K = U\Sigma V^T$$



SVD truncation

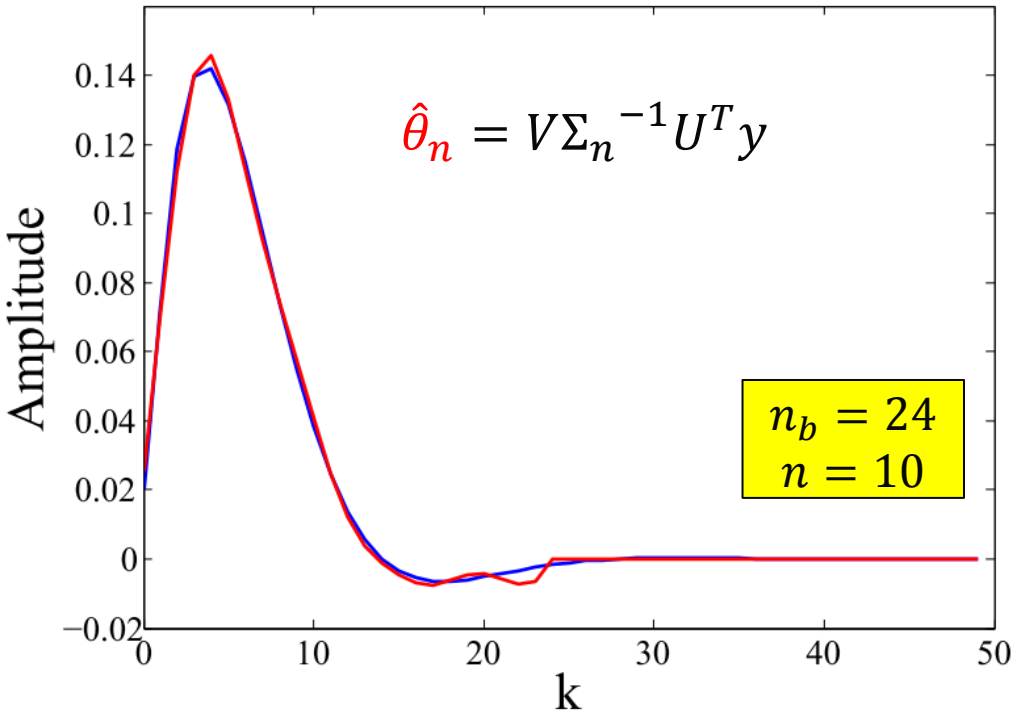
$$\hat{\theta} = (K^T K)^{-1} K^T y = V \Sigma^{-1} U^T y$$

low-rank solution $\hat{\theta}_n = V {\Sigma_n}^{-1} U^T y$

$$\Sigma_n{}^{-1} = \begin{bmatrix} \frac{1}{\sigma_1} & & & & 0 \\ & \ddots & & & \\ & & \frac{1}{\sigma_n} & & \\ & & & 0 & \\ 0 & & & & \ddots \\ & & & & & 0 \end{bmatrix}$$

n_b , n

Truncated estimate



Impulse response modeling

1. **Low-rank solution**
   - SVD truncation

2. **Regularized solution**
   - Empirical Bayes

A different approach: regularization

$$\hat{\theta}_{\text{reg}} = \arg\min_{\theta} \|y - K\theta\|^2 + \lambda\theta^T R\theta$$
$$= (K^T K + \lambda R)^{-1} K^T y$$

$$\lambda R = \sigma^2 \Pi^{-1}$$

- λR: "amount" of regularization
- λR: prior knowledge

$$\begin{cases} \sigma^2 & \text{noise variance} \\ \Pi & \text{covariance of } \theta \end{cases}$$

n_b

Parametrization of Π for FIR models

$$\Pi_{k,j} = C\ \rho^{|k-j|}\ \alpha^{(k+j)/2}$$

smoothness (correlation)

exponential decay (diagonal)

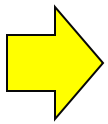
'DC' kernel

$C \geq 0, 0 \leq \alpha \leq 1, |\rho| \leq 1$

$$\rho = \alpha^{1/2} \longrightarrow \Pi_{k,j} = C \min(\alpha^k, \alpha^j)$$

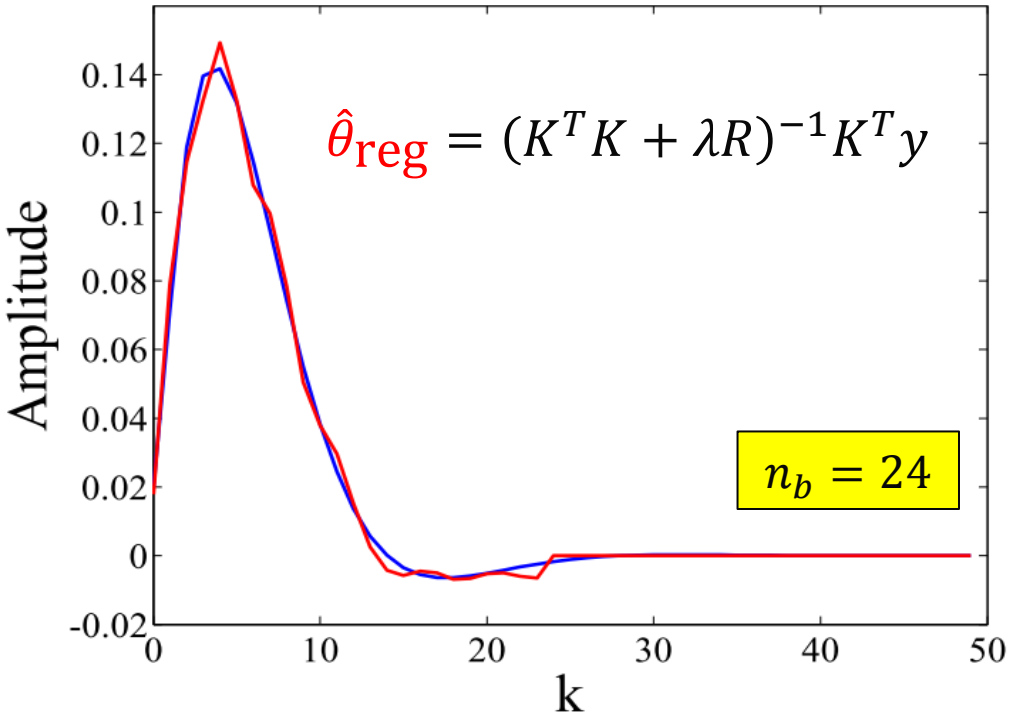
'TC' kernel

hyperparameters estimated by Marginal Likelihood Maximization



$\boldsymbol{\lambda, R}$

Regularized estimate



Impulse response modeling

1. **Low-rank solution**
   - SVD truncation



LINK



2. **Regularized solution**
   - Empirical Bayes

SVD truncation as regularization task

$$\hat{\theta}_{\text{reg}} = (K^T K + \lambda R)^{-1} K^T y$$

$$\lambda R = \lambda V D V^T$$

$$D = \begin{bmatrix} \overbrace{\begin{matrix} 0 & & \\ & \ddots & \\ & & 0 \end{matrix}}^{n} & & \\ & \underbrace{\begin{matrix} 1 & & \\ & \ddots & \\ & & 1 \end{matrix}}_{n_b - n} & \\ 0 & & \end{bmatrix}$$

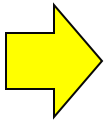
$$\hat{\theta}_{\text{reg}} = (V(\Sigma^2 + \lambda D)V^T)^{-1} V \Sigma U^T y = V(\Sigma^2 + \lambda D)^{-1} \Sigma U^T y$$

for $\lambda \gg 1$: $\quad (\Sigma^2 + \lambda D)^{-1} \Sigma \approx {\Sigma_n}^{-1}$

$$\boxed{\hat{\theta}_{\text{reg}} = V {\Sigma_n}^{-1} U^T y = \hat{\theta}_n}$$

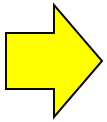
Prior knowledge

Information about the excitation: coloring of the input



1. **Low-rank solution**
   - SVD truncation

Information about the system: exponential decay and smoothness



2. **Regularized solution**
   - Empirical Bayes

Combining the two approaches

low-rank truncated solution: $\hat{\theta} = \tilde{V}\tilde{\theta}$

first n columns

n-dimensional

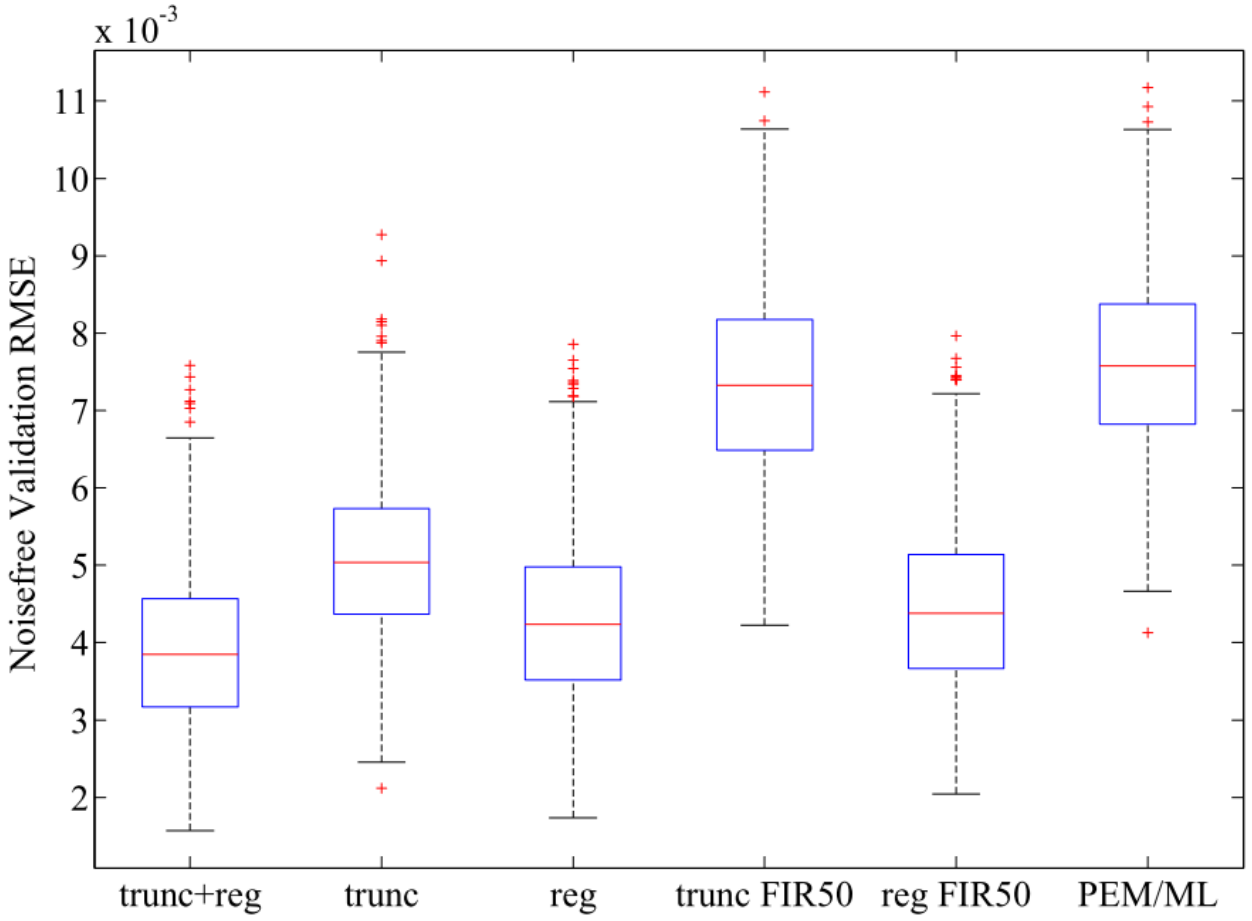
$$\tilde{\theta} = \left(\tilde{K}^T\tilde{K} + \lambda\tilde{R}\right)^{-1}\tilde{K}^T y$$

$$\tilde{K} = K\tilde{V} = U\Sigma V^T\tilde{V}$$

$$\tilde{R} = \tilde{V}^T R\tilde{V}$$



Monte Carlo simulation



Conclusion

- FIR modeling
  - Colored noise excitation

1. **SVD truncation**
   - Prior: input signal



2. **Bayesian regularization**
   - Prior: system response

- Outperform PEM/ML
  - Validation error: -50%

- Combination of methods beneficial

Thank you for your attention!

Any questions?

Combining regularization and SVD truncation for impulse response modeling

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