Data-driven robust receding horizon fault estimation

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Outline

- Problem formulation
- Predictor-based receding horizon fault estimation
- Data-driven nominal fault estimator
- Data-driven robust fault estimator
- Simulation examples
- Summary

Problem formulation

- Fault estimation for unknown LTI systems

$$\begin{cases} \xi(k+1) = A\xi(k) + Bu(k) + Ef(k) + Fw(k) \\ y(k) = C\xi(k) + Du(k) + Gf(k) + v(k) \end{cases}$$

- Actuator/sensor faults as additive unknown inputs

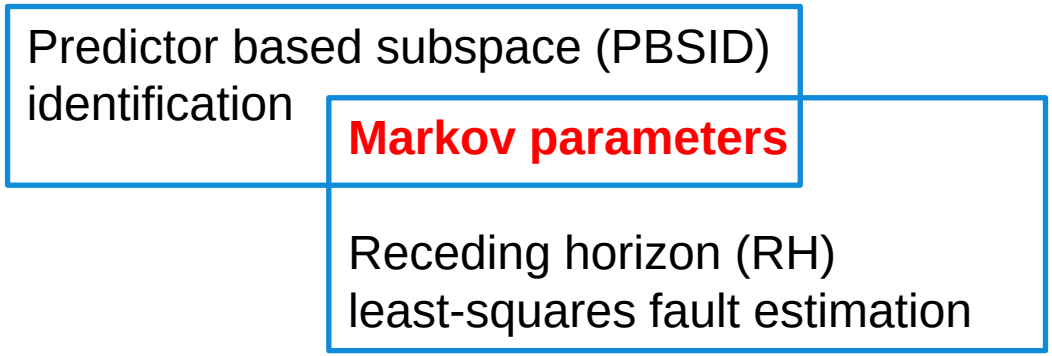
$$\begin{cases} E = 0_{n_x \times 1}, G = I^{[i]} & \text{for faults of the } i^{\text{th}} \text{ sensor} \\ E = B^{[i]}, G = D^{[i]} & \text{for faults of the } i^{\text{th}} \text{ actuator} \end{cases}$$

- Only I/O data collected from the fault-free condition are available

Difficulties and basic idea

- Data-driven unknown input estimation:
  - Unbiased fault estimation in the presence of unknown initial state
  - Robustness to identification errors

- Basic idea:



Predictor-based RH fault estimator

- Markov parameters of Predictor form:

$$\begin{cases} x(k+1) = \Phi x(k) + \tilde{B}u(k) + \tilde{E}f(k) + Ky(k) \\ y(k) = Cx(k) + Du(k) + Gf(k) + e(k) \end{cases}$$

$$\left\{H_i^u\right\}:\left(\Phi,\tilde{B},C,D\right)$$
$$\left\{H_i^y\right\}:\left(\Phi,K,C,0\right)$$
$$\left\{H_i^f\right\}:\left(\Phi,\tilde{E},C,G\right)$$

- Extended output equation in a sliding window [k-L+1, k]

$$\mathbf{y}_{k,L} = \mathcal{O}_L x(k_0) + \mathbf{H}^o_{L,m}\zeta_m + \mathbf{T}^f_{L,\tau}\mathbf{f}_{k-\tau,L-\tau} + \mathbf{T}^y_L\mathbf{y}_{k,L} + \mathbf{T}^u_L\mathbf{u}_{k,L} + \mathbf{e}_{k,L}$$

- Residual generation

  τ denotes the relative degree of (A, E, C, G)

$$\mathbf{r}_{k,L} = \mathbf{y}_{k,L} - \mathbf{T}^y_L\mathbf{y}_{k,L} - \mathbf{T}^u_L\mathbf{u}_{k,L} = \underbrace{\begin{bmatrix}\mathbf{H}^o_{L,m} & \mathbf{T}^f_{L,\tau}\end{bmatrix}}_{\Psi_{L,\tau}} \cdot \underbrace{\begin{bmatrix}\zeta_m \\ \mathbf{f}_{k-\tau,L-\tau}\end{bmatrix}}_{\mathbf{f}^a_{k-\tau,L-\tau}} + \mathbf{e}_{k,L}$$

Estimation properties

- Least-squares (LS) fault estimation

$$\min_{\mathbf{f}^a_{k-\tau,L-\tau}} \left\| \mathbf{r}_{k,L} - \Psi_{L,\tau} \mathbf{f}^a_{k-\tau,L-\tau} \right\|^2_{\Sigma^{-1}_{e,L}}$$

$$\begin{cases} \hat{\mathbf{f}}^a_{k-\tau,L-\tau} = \left(\Psi^{\mathrm{T}}_{L,\tau} \Sigma^{-1}_{e,L} \Psi_{L,\tau} \right)^{(1)} \Psi^{\mathrm{T}}_{L,\tau} \Sigma^{-1}_{e,L} \mathbf{r}_{k,L} \\ \hat{f}\left(k-\tau\right) = \mathcal{I}_{n_f} \cdot \hat{\bar{\mathbf{f}}}_{k-\tau,L-\tau}, \ \mathcal{I}_{n_f} = \begin{bmatrix} 0 & I_{n_f} \end{bmatrix} \end{cases}$$

$$\hat{\mathbf{f}}^a_{k-\tau,L-\tau} = \begin{bmatrix} \hat{\xi}_m \\ \hat{f}\left(k-L+1\right) \\ \vdots \\ \hat{f}\left(k-\tau\right) \end{bmatrix}$$

- Unbiasedness condition:
  - $\hat{f}(k-\tau)$ is unbiased for all $L \geq n+\tau$ if and only if $\left(A, E, O_{\tau+1}, \mathbf{H}^f_{s,\tau}\right)$ has no transmission zero
  - $\hat{f}(k-\tau)$ is asymptotically unbiased for $L \to \infty$ if and only if all transmission zeros of $\left(A, E, O_{\tau+1}, \mathbf{H}^f_{s,\tau}\right)$ are stable
- $\hat{f}(k-\tau)$ is independent of predictor gain K

Data-driven nominal fault estimator

- PBSID

$$\hat{\Xi} = \mathbf{Y}_{\mathrm{id}}\mathbf{Z}_{\mathrm{id}}^{-} \qquad \hat{\Xi} = \begin{bmatrix} \hat{H}_p^u & \hat{H}_p^y & \cdots & \hat{H}_1^u & \hat{H}_1^y & \hat{H}_0^u \end{bmatrix}$$

$$\hat{\Sigma}_e = \mathrm{cov}\left(\mathbf{Y}_{\mathrm{id}} - \hat{\Xi}\mathbf{Z}_{\mathrm{id}}\right)$$

- Residual generation

$$\hat{\mathbf{r}}_{k,L} = \mathbf{y}_{k,L} - \hat{\mathbf{T}}_L^y \mathbf{y}_{k,L} - \hat{\mathbf{T}}_L^u \mathbf{u}_{k,L} \approx \underbrace{\begin{bmatrix} \hat{\mathbf{H}}_{L,m}^o & \hat{\mathbf{T}}_{L,\tau}^f \end{bmatrix}}_{\hat{\Psi}_{L,\tau}} \cdot \underbrace{\begin{bmatrix} \zeta_m \\ \mathbf{f}_{k-\tau,L-\tau} \end{bmatrix}}_{\mathbf{f}_{k-\tau,L-\tau}^a} + \mathbf{e}_{k,L}$$

- Least-squares (LS) fault estimation

$$\min_{\mathbf{f}_{k-\tau,L-\tau}^a} \left\| \hat{\mathbf{r}}_{k,L} - \hat{\Psi}_{L,\tau} \mathbf{f}_{k-\tau,L-\tau}^a \right\|_{\hat{\Sigma}_{e,L}^{-1}}^2$$

$$\begin{cases} \hat{\mathbf{f}}_{k-\tau,L-\tau}^a = \left(\hat{\Psi}_{L,\tau}^{\mathrm{T}} \Sigma_{e,L}^{-1} \hat{\Psi}_{L,\tau} \right)^{(1)} \hat{\Psi}_{L,\tau}^{\mathrm{T}} \Sigma_{e,L}^{-1} \mathbf{r}_{k,L} \\ \hat{f}\left(k-\tau\right) = \mathcal{I}_{n_f} \cdot \hat{\mathbf{f}}_{k-\tau,L-\tau}, \ \mathcal{I}_{n_f} = \begin{bmatrix} 0 & I_{n_f} \end{bmatrix} \end{cases}$$

Estimation error analysis

- Identification errors in PBSID

$$\begin{aligned}\hat{\Xi} &= \begin{bmatrix} \hat{H}_p^u & \hat{H}_p^y & \cdots & \hat{H}_1^u & \hat{H}_1^y & \hat{H}_0^u \end{bmatrix} \\ &= \Xi + C\Phi^p \mathbf{X}_{\mathrm{id}} \mathbf{Z}_{\mathrm{id}}^- \\ &\quad + \mathbf{E}_{\mathrm{id}} \underbrace{\begin{bmatrix} \hat{M}_p^u & \hat{M}_p^y & \cdots & \hat{M}_1^u & \hat{M}_1^y & \hat{M}_0^u \end{bmatrix}}_{\mathbf{Z}_{\mathrm{id}}^-}\end{aligned}$$

- Estimation error

$$\begin{aligned}\Delta f(k-\tau) &= \mathcal{G}\hat{\mathbf{r}}_{k,L} - \mathcal{I}_{n_f} \cdot \overline{\mathbf{f}}_{k-\tau,L-\tau}^a \\ &= \underbrace{\left(\mathcal{G}\hat{\Psi}_{L,\tau} - \mathcal{G}\overline{\mathbf{E}}_{\mathrm{id}}\overline{\mathbf{M}}_{\Psi} - \mathcal{I}_{n_f}\right)}_{\mathcal{T}_f(\mathcal{G})} \overline{\mathbf{f}}_{k-\tau,L-\tau}^a - \underbrace{\mathcal{G}\overline{\mathbf{E}}_{\mathrm{id}}\overline{\mathbf{M}}_L^z}_{\mathcal{T}_z(\mathcal{G})} \mathbf{z}_{k,L} + \mathcal{G}\mathbf{e}_{k,L}\end{aligned}$$

Data-driven robust fault estimator

- Estimation error analysis

$$\begin{aligned}\Delta f(k-\tau) &= \mathcal{G}\hat{\mathbf{r}}_{k,L} - \mathcal{I}_{n_f}\cdot\overline{\mathbf{f}}^{a}_{k-\tau,L-\tau} \\ &= \underbrace{\left(\mathcal{G}\hat{\Psi}_{L,\tau} - \mathcal{G}\bar{\mathbf{E}}_{\mathrm{id}}\bar{\mathbf{M}}_{\Psi} - \mathcal{I}_{n_f}\right)}_{\mathcal{T}_f(\mathcal{G})}\overline{\mathbf{f}}^{a}_{k-\tau,L-\tau} - \underbrace{\mathcal{G}\bar{\mathbf{E}}_{\mathrm{id}}\bar{\mathbf{M}}^{z}_{L}}_{\mathcal{T}_z(\mathcal{G})}\mathbf{z}_{k,L} + \mathcal{G}\mathbf{e}_{k,L}\end{aligned}$$

- Mixed $\mathcal{H}_2/\mathcal{H}_\infty$ problem

$$\begin{aligned}&\mathcal{G}_{\mathrm{r,off}} = \arg\min_{\mathcal{G}} \operatorname{tr}\left(\mathcal{G}\Sigma_{e,L}\mathcal{G}^{\mathrm{T}}\right) \\ &\text{s.t. } \bar{\mathbb{E}}\left\|\mathcal{T}_f\left(\mathcal{G}\right)\right\|_\infty \le \gamma_f, \bar{\mathbb{E}}\left\|\mathcal{T}_z\left(\mathcal{G}\right)\right\|_\infty \le \gamma_z \qquad 0 < \gamma_f < 1\end{aligned}$$

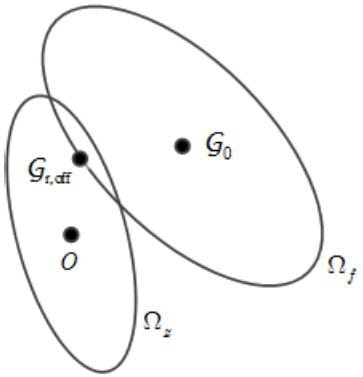
$$\begin{aligned}&\mathcal{G}_{\mathrm{r,off}} = \arg\min_{\mathcal{G}} \operatorname{tr}\left(\mathcal{G}\Sigma_{e,L}\mathcal{G}^{\mathrm{T}}\right) \\ &\text{s.t.}\begin{bmatrix}\mathcal{G} & \mathcal{I}_{n_f}\end{bmatrix}\begin{bmatrix}\Pi_f & -\hat{\Psi}_{L,\tau} \\ -\hat{\Psi}^{\mathrm{T}}_{L,\tau} & I_{n_f}\end{bmatrix}\begin{bmatrix}\mathcal{G}^{\mathrm{T}} \\ \mathcal{I}^{\mathrm{T}}_{n_f}\end{bmatrix} \le \gamma_f^2 I \\ &\mathcal{G}\Pi_z\mathcal{G}^{\mathrm{T}} \le \gamma_z^2 I\end{aligned}$$

Geometric interpretation

$$\mathcal{G}_{\mathrm{r,off}} = \arg\min_{\mathcal{G}} \operatorname{tr}\left(\mathcal{G}\Sigma_{e,L}\mathcal{G}^{\mathrm{T}}\right)$$

$$\text{s.t.} \begin{bmatrix} \mathcal{G} & \mathcal{I}_{n_f} \end{bmatrix} \begin{bmatrix} \Pi_f & -\hat{\Psi}_{L,\tau} \\ -\hat{\Psi}_{L,\tau}^{\mathrm{T}} & I_{n_f} \end{bmatrix} \begin{bmatrix} \mathcal{G}^{\mathrm{T}} \\ \mathcal{I}_{n_f}^{\mathrm{T}} \end{bmatrix} \leq \gamma_f^2 I \qquad 0 < \gamma_f < 1$$

$$\mathcal{G}\Pi_z\mathcal{G}^{\mathrm{T}} \leq \gamma_z^2 I$$

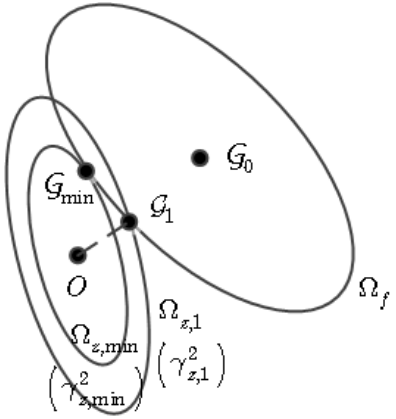


Geometric interpretation

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$$\mathcal{G}\Pi_z\mathcal{G}^{\mathrm{T}} \leq \gamma_z^2 I$$



	Averaged bias $\bar{\mathbb{E}}(\mathcal{T}_f(\mathcal{G}_{\mathrm{r,off}}))\bar{\mathbf{f}}_{k-\tau,L-\tau}^{a}$	**Averaged bias** $\bar{\mathbb{E}}(\mathcal{T}_z(\mathcal{G}_{\mathrm{r,off}}))\mathbf{z}_{k,L}$	**Variance** $\operatorname{tr}\left(\mathcal{G}_{\mathrm{r,off}}\Sigma_{e,L}\mathcal{G}_{\mathrm{r,off}}^{\mathrm{T}}\right)$
$\gamma_z^2 \nearrow$	constant	$\nearrow$ or constant	$\searrow$ or constant
$\gamma_f^2 \nearrow$	$\nearrow$	$\searrow$	$\searrow$

Enhance robustness with online optimization

- Estimation error analysis

$$\Delta f(k-\tau) = \underbrace{\left(\mathcal{G}\hat{\Psi}_{L,\tau} - \mathcal{G}\bar{\mathbf{E}}_{\mathrm{id}}\bar{\mathbf{M}}_{\Psi} - \mathcal{I}_{n_f}\right)}_{\mathcal{T}_f(\mathcal{G})}\bar{\mathbf{f}}^{a}_{k-\tau,L-\tau} - \underbrace{\mathcal{G}\bar{\mathbf{E}}_{\mathrm{id}}\bar{\mathbf{M}}^{z}_{L}}_{\mathcal{T}_z(\mathcal{G})}\mathbf{z}_{k,L} + \mathcal{G}\mathbf{e}_{k,L}$$
$$= \mathcal{T}_f\left(\mathcal{G}\right)\bar{\mathbf{f}}^{a}_{k-\tau,L-\tau} - \mathcal{G}\Gamma_{k,L}\cdot\mathrm{vec}\left(\mathbf{E}_{\mathrm{id}}\right) + \mathcal{G}\mathbf{e}_{k,L}$$

- Online $\mathcal{H}_2/\mathcal{H}_\infty$ problem

$$\mathcal{G}_{\mathrm{r,on}} = \arg\min_{\mathcal{G}} \mathrm{tr}\left(\mathcal{G}\Sigma_{e,L}\mathcal{G}^{\mathrm{T}} + \mathcal{G}\Gamma_{k,L}\left(I_N \otimes \Sigma_e\right)\Gamma^{\mathrm{T}}_{k,L}\mathcal{G}^{\mathrm{T}}\right)$$
$$\text{s.t. } \bar{\mathbb{E}}\left\|\mathcal{T}_f\left(\mathcal{G}\right)\right\|_\infty \le \gamma_f, \left\|\mathcal{G}\Gamma_{k,L}\right\|_\infty \le \gamma_e$$

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$$\text{s.t.} \begin{bmatrix}\mathcal{G} & \mathcal{I}_{n_f}\end{bmatrix}\begin{bmatrix}\Pi_f & -\hat{\Psi}_{L,\tau} \\ -\hat{\Psi}^{\mathrm{T}}_{L,\tau} & I_{n_f}\end{bmatrix}\begin{bmatrix}\mathcal{G}^{\mathrm{T}} \\ \mathcal{I}^{\mathrm{T}}_{n_f}\end{bmatrix} \le \gamma_f^2 I$$
$$\mathcal{G}\Gamma_{k,L}\Gamma^{\mathrm{T}}_{k,L}\mathcal{G}^{\mathrm{T}} \le \gamma_e^2$$

Simulation settings

- Linearized vertical take-off and landing model
  - 2 inputs, 4 outputs
- Empirical stabilizing controller

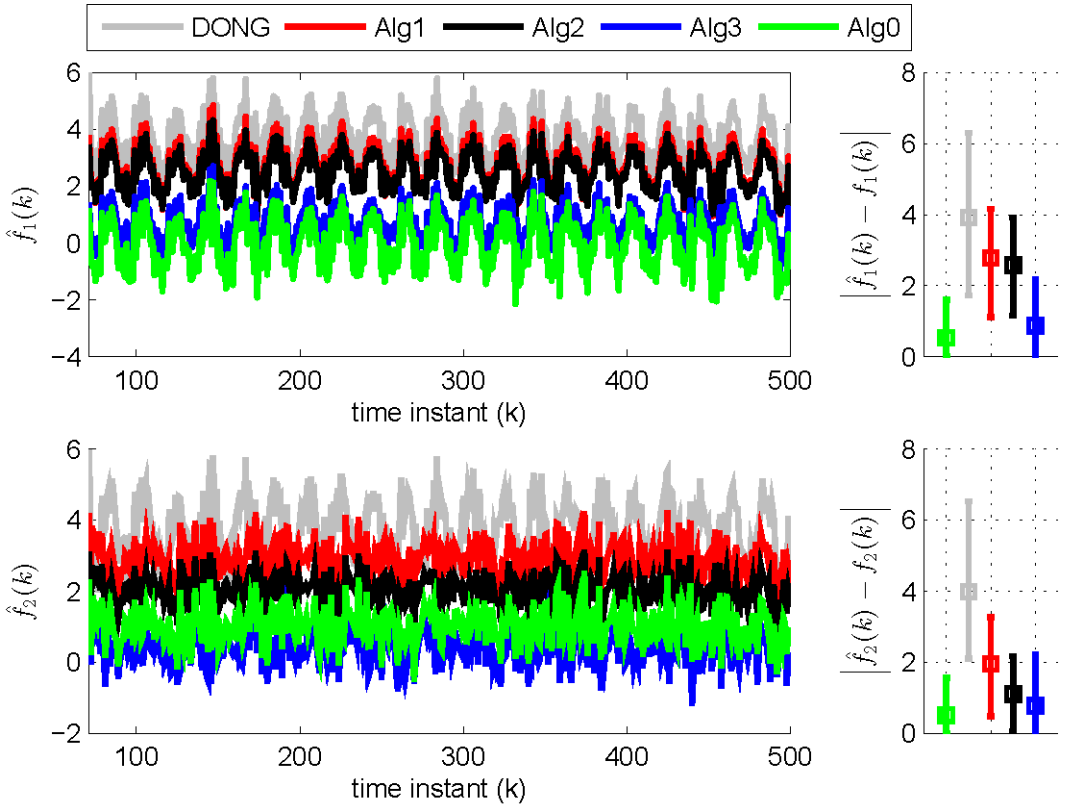
$$u(k) = -\begin{bmatrix} 0 & 0 & -0.5 & 0 \\ 0 & 0 & -0.1 & -0.1 \end{bmatrix} \cdot y(k) + \eta(k)$$

- Fault cases:
  - Actuator faults: $E = B, G = D$
  - Sensor faults: $E = 0_{4\times 2}, G = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$

$$f(k) = \begin{cases} \begin{bmatrix} 0 & 0 \end{bmatrix}^{\mathrm{T}}, & 0 \le k \le 50, \\ \begin{bmatrix} \sin\left(0.1\pi k\right) & 1 \end{bmatrix}^{\mathrm{T}}, & k > 50. \end{cases}$$

Simulation results

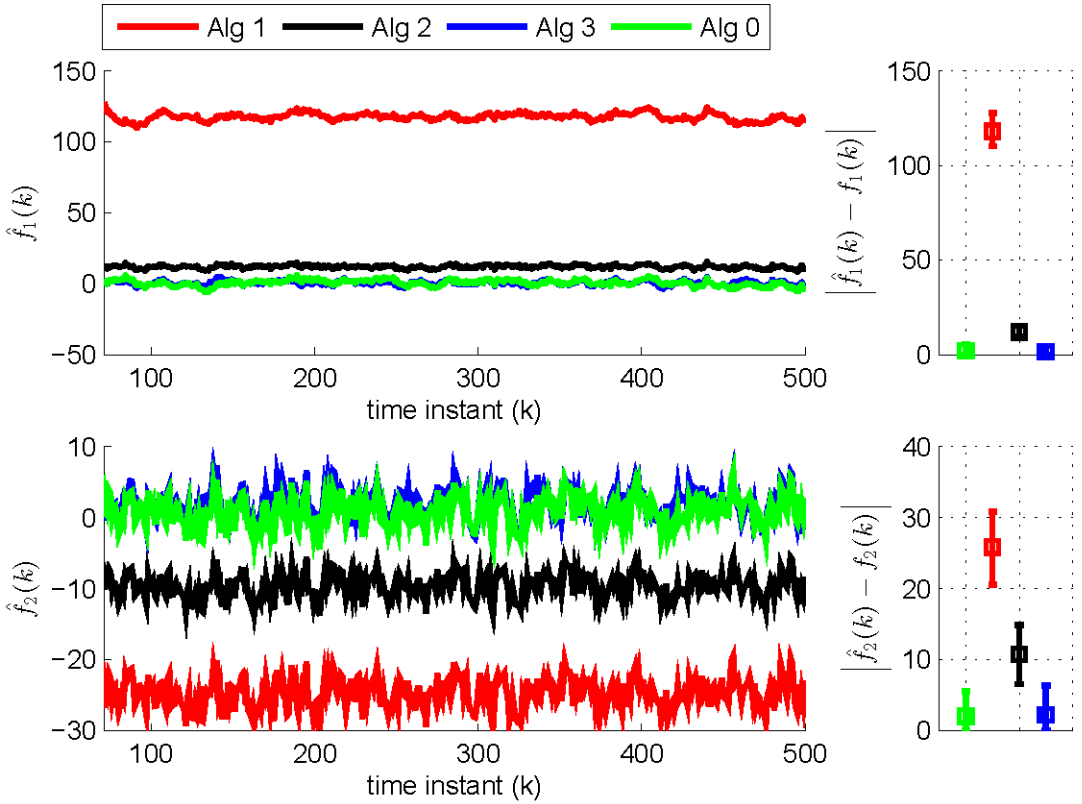
- $\eta(k) = 15$, actuator faults



- **Alg0:**
  accurate Markov parameters
- **Dong:**
  Dong and Verhaegen, 2012
- **Alg1:**
  data-driven nominal estimator
- **Alg2:**
  data-driven robust estimator
- **Alg3:**
  data-driven robust estimator with online optimization

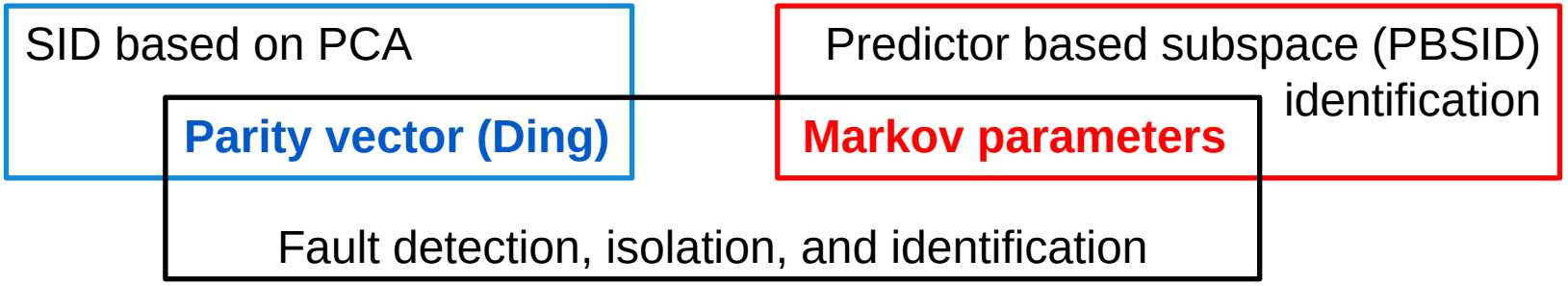
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Summary



- Unbiasedness condition for RH fault estimator based on Markov parameters
- Robustness to identification errors using $\mathcal{H}_2/\mathcal{H}_\infty$ optimization
  - Offline design
  - Online optimization
  - Parameter tuning using geometric interpretation

Thanks for your attention!

Questions or suggestions?

Simulation results: parameter tuning

Alg2: data-driven robust fault estimator

γ_f^2 ↗	**Averaged bias↗** $\overline{\mathbb{E}}(\mathcal{T}_f(\mathcal{G}_{\text{r,off}}))\bar{\mathbf{f}}^a_{k-\tau,L-\tau}$	**Averaged bias↘** $\overline{\mathbb{E}}(\mathcal{T}_z(\mathcal{G}_{\text{r,off}}))\mathbf{z}_{k,L}$	**Variance↘** $\text{tr}\left(\mathcal{G}_{\text{r,off}}\Sigma_{e,L}\mathcal{G}_{\text{r,off}}^{\text{T}}\right)$

